## SECTION-A

**1. Find HCF of 196 and 38220 using Euclid's divisionAlgorithm.** $\left[2\frac{1}{2}\right]$

Ans. 38220 > 196, we apply the division lemma to 38220 and 196, to get

38220 = 196 × 195 + 0

The remainder now becomes zero, so our procedure stops.

Since the divisor at this stage is 196. So the HCF of 38220 and 196 is 196.

**2. If HCF (306, 657) = 9 then find the LCM (306, 657).** $\left[2\frac{1}{2}\right]$

Ans. We know that

$LCM\ (a, b) \times HCF\ (a, b) = a \times b$

$\Rightarrow LCM\ (a, b) = \frac{a \times b}{HCF\,(a,b)}$

$\Rightarrow LCM\ (306, 657) = \frac{306 \times 657}{9}$

$\Rightarrow LCM(306, 657) = 22338.$

**3. Solve the following pair of Linear equation** $\left[2\frac{1}{2}\right]$

$s - t = 3$

${}^{s}/_{3} + {}^{t}/_{2} = 6$

Ans. Given pair of linear equations

$s - t = 3$ ……………(i)

$\frac{s}{3} - \frac{t}{2} = 6$ ……………(ii)

From equation (i) , we get

$s = t + 3$ ……………(iii)

Substituting this value s in equation (ii), we get

$\frac{t+3}{3} + \frac{t}{2} = 6$

$\Rightarrow 2(t + 3) + 3t = 36$

$\Rightarrow 5t + 6 = 36 \Rightarrow \quad t = 6.$

Substituting the value t in equation (iii), we get

$$s + 6 + 3 = 9$$

Hence, $s = 9, t = 6.$

**4. Find the roots of quadratic $6x^2 - x - 2 = 0$.** $\left[2\frac{1}{2}\right]$

Ans. $6x^2 - x - 2 = 0$

By spitting the middle term $-x$ as $-4x + 3x$.

$6x^2 - 4x + 3x - 2 = 0$

$2x(3x - 2) + 1(3x - 2) = 0$

$(3x - 2)(2x + 1) = 0$

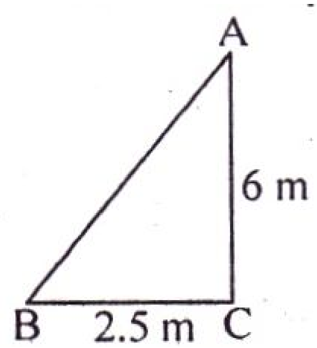
$3x - 2 = 0$ $\quad$ $2x + 1 = 0$

$3x = 2$ $\quad$ $2x = -1$

$x = \frac{2}{3}$ $\quad$ $x = \frac{-1}{2}$

**5. A ladder is placed against a wall such that its foot is at a distance of 2.5 m from the wall and its top reaches a window 6 mabove the ground. Find the height (length) of the ladder.** $\left[2\frac{1}{2}\right]$

Ans. Let's assume that AB is a ladder and CA is a wall. The window is on a point A.




In figure, BC = 2.5 m, CA = 6m

In right $\Delta ABC$,

$AB^2 = BC^2 + CA^2$

$AB^2 = (2.5)^2 + (6)^2$

$AB^2$ = 6.25 + 36 = 42.25.

$\Rightarrow AB$ = 6.5 $m$

Hence, length of the ladder = 6.5m

**6.** **Find the point on $x$ −axis which is equidistant from $(2, -5)$ $and$ $(-2, 9)$.** $\left[2\frac{1}{2}\right]$

Ans. Let the coordinates of $A(2, -5)$ $and$ $B(-2, 9)$ are given points.

and $P(x, 0)$ be the required point on the $x$ −axis such that

PA = PB

$$PA = \sqrt{(2-x)^2 + (-5-0)^2}$$

$$= \sqrt{4 + x^2 - 4x + 25}$$

$$= \sqrt{x^2 - 4x + 29}$$

and $PB = \sqrt{(-2-x)^2 + (9-0)^2}$

$$= \sqrt{4 + x^2 + 4x + 81}$$

$$= \sqrt{x^2 + 4x + 85}$$

It is given that PA = PB

$\Rightarrow \quad \sqrt{x^2 - 4x + 29} = \sqrt{x^2 + 4x + 85}$

$\Rightarrow \quad x^2 - 4x + 29 = x^2 + 4x + 85$

$\Rightarrow \quad -4x + 29 = 4x + 85$

$\Rightarrow \quad -4x - 4x = 85 - 29$

$\Rightarrow \quad -8x = 56$

$\Rightarrow \quad x = -7$

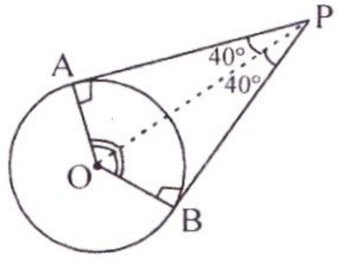
Hence, point on $x - axis = -7$

Therefore, the point equidistant from the given points is (-7, 0).

**7.** **If tangents PA and PB from a point P to a circle with centre O are inclined to each other at angle $80^0$, then find the value of $\angle POA$.** $\left[2\frac{1}{2}\right]$

Ans. From the fig., we have

$\Delta AOP \cong \Delta OBP$ $(\because By\ SSS\ congruence)$




$\Rightarrow \angle POA = \angle POB = \frac{1}{2}\angle AOB$ ……………………….(i)

We know that

$\angle AOB + \angle APB = 180^0$

$\Rightarrow \angle AOB + 80^0 = 180^0$

$\Rightarrow \angle AOB = 180^0 - 80^0 = 100^0$ ………………………..(ii)

From (i) and (ii), we get.,

$\angle POA = \frac{1}{2} \times 100 = 50^0$

Hence, $\angle POA = 50^0$.

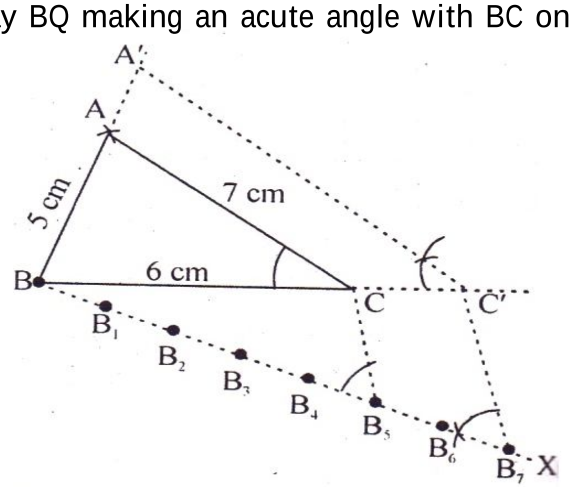
**8. Construct a triangle with sides 5 cm, 6 cm, 7 cm amd then another triangle whose sides are $\frac{7}{5}$ of the corresponding sides of first triangle.** $\left[2\frac{1}{2}\right]$

Ans. We are given the sides of triangle 5 cm, 6 cm and 7 cm and we are required to construct another triangle whose sides are $\frac{7}{5}$ of the corresponding sides of first triangle.

**Steps of construction:**

(i) Construct a triangle ABC in which AB = 5 cm, BC = 6 cm and CA =7cm.

(ii) Draw any ray BQ making an acute angle with BC on the side opposite to the vertex A.



(iii) Marks 7 points (the greater of 7 and 5 in 7/5) $B_1, B_2, B_3, B_4, B_5, B_6$ $and$ $B_7$ So that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5 = B_5B_6 = B_6B_7$.

(iv) Join $B_5C$.

(v) Through $B_7$, draw $B_7C'||B_5C$ to intersect BC at C'.

(vi) Through C', draw C'A'|| CA to intersect BA at A' Now $\Delta A'BC'$ is the required triangle whose sides are 7/5 times the corresponding sides of the $\Delta ABC$.

Justification

$\Delta ABC \sim A'BC'$ (By AA similarity criteria)

Therefore, $\frac{BA'}{BA} = \frac{BC'}{BC} = \frac{A'C'}{AC}$

But $\frac{BC'}{BC} = \frac{7}{5}$ $\left(\because \frac{BB_7}{BB_5} = \frac{BC'}{BC} = \frac{7}{5}\right)$

So, $\frac{BA'}{BA} = \frac{BC'}{BC} = \frac{A'C'}{AC} = \frac{7}{5}$

**9.** **Gopi buys a fish from a shop. If there are 5 male and 8 female fishes in the tank what will be probability of a fish taken out randomly to be a male fish?** $\left[2\frac{1}{2}\right]$

Ans. Number of male fish = 5

Number of female fish = 8

Then total number of fishes in the tank =5+8=13

$\therefore$ Probability of getting a male fish = $\frac{\text{possible outcomes}}{\text{Total outcomes}} = \frac{5}{13}$.

**10.** **One card is drawn from a well shuffled deck of 52 cards. Find the probability of getting a king of red colour.** $\left[2\frac{1}{2}\right]$

Ans. There are two red kings, (one each of diamond and heart)

So, number of favourable outsomes = 2

$\therefore$ Probability off getting king of red colour $= \frac{2}{52} = \frac{1}{26}$

## SECTION-B

**11.** **Divide** $(3x^4 + 5x^3 - 7x^2 + 2x + 2)$ $by$ $(x^2 + 3x + 1)$. $\left[3\frac{1}{2}\right]$

Ans. Here, dividend and divisor are in the standard form.

$$
\begin{array}{r|l}
 & 3x^2 - 4x + 2 \\
\hline
x^2 + 3x + 1 & 3x^4 + 5x^3 - 7x^2 + 2x + 2 \\
 & 3x^4 + 9x^3 + 3x^2 \\
 & - \quad - \quad - \\
\hline
 & -4x^3 - 10x^2 + 2x + 2 \\
 & -4x^3 - 12x^2 - 4x \\
 & + \quad + \quad + \\
\hline
 & 2x^2 + 6x + 2 \\
 & 2x^2 + 6x + 2 \\
 & - \quad - \quad - \\
\hline
 & 0
\end{array}
$$

Since, remainder is zero, hence $x^2 + 3x + 1$ is a factor of $3x^4 + 5x^3 - 7x^2 + 2x + 2$.

**12.** **Solve the pair of equations graphically:** $\left[3\frac{1}{2}\right]$

$$\boldsymbol{x + 3y = 6}$$

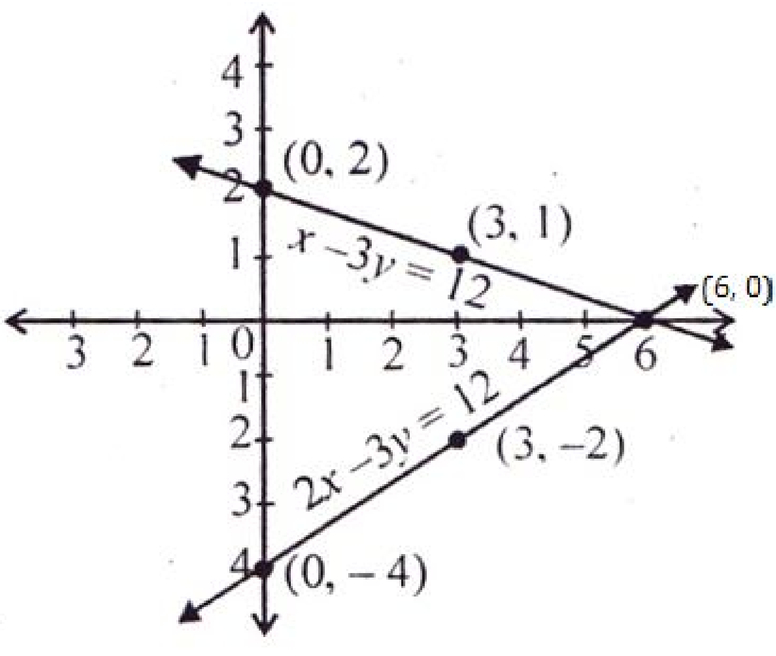
$$\boldsymbol{2x - 3y = 12}$$

Ans. **Graphically representation:**

From equation (i), we get

| $x$ | 6 | 3 | 0 |
|---|---|---|---|
| $y$ | 0 | 1 | 2 |

$x + 6 - 3y$



From equation (ii), we get

$2x - 3y = 12$

$\Rightarrow \quad x = \frac{12+3y}{2}$

| $x$ | 6 | 3 | 0 |
|---|---|---|---|
| $y$ | 0 | $-2$ | $-4$ |

From the graph we find that lines are intersecting lines and have unique solution.

Hence, $x = 6, y = 0$.

**13. How many multiples of 4 lie between 10 and 250?** $\left[3\frac{1}{2}\right]$

Ans. The multiples of 4 between 10 and 250 are 12, 16, 20, 24 ……………………, 248.

So, the above sequence from an A.P.

Let these number = n.

$a_1 = 12,$

$d = a_2 - a_1$

$= 16 - 12 = 4$

$a_n = 248.$

We know that

$a_n = a + (n-1)d$

$\Rightarrow 248 = 12 + (n-1)4$

$\Rightarrow 248 = 12 + 4n - 4$

$\Rightarrow 4n = 248 - 8$

$\Rightarrow n = \frac{240}{4}$

$\Rightarrow n = 60$

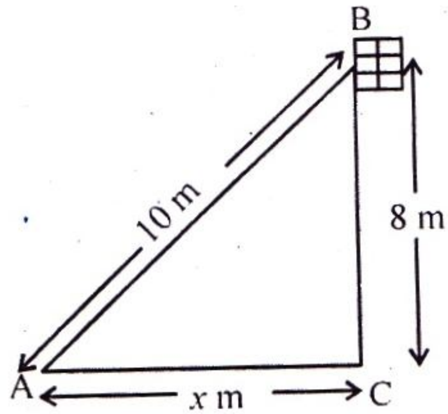
Hence, there are 60 terms lies multiple of 4 betwene 10 and 250.

**14. A ladder 10 m long reaches a window 8m above the ground. Find the distance of the foot of ladder from base of wall.** $\left[3\frac{1}{2}\right]$

Ans. Length of the ladder = 10 m

The height between a window and ground = 8 m

Let's assume the distance between foot of the ladder and base of the ball = $xm$




In right $\Delta ABC$,

$(AB)^2 = (BC)^2 + (AC)^2$ (Pythagoras Theorem)

$\Rightarrow (10)^2 = (8)^2 + (x)^2$

$\Rightarrow (x)^2 = (10)^2 - (8)^2$

$\Rightarrow x^2 = 100 - 64 = 36$

$\Rightarrow x = 6$

Hence, the distance between foot of the ladder and base of the wall = 6m

**15.** **Prove that :** $(\boldsymbol{cosec\,\theta} - \mathbf{cot\,\theta})^2 = \frac{\mathbf{1-cos\,\theta}}{\mathbf{1+cos\,\theta}}$. $\left[3\frac{1}{2}\right]$

Ans. $L.H.S = (cosec\,\theta - \cot\theta)^2$

$= \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2 = \left(\frac{1-\cos\theta}{\sin\theta}\right)^2$

$= \frac{(1-\cos\theta)^2}{sin^2\theta} = \frac{(1-\cos\theta)^2}{1-cos^2\theta}$ $[\because sin^2\theta = 1 - cos^2\theta]$

$= \frac{(1-\cos\theta)^2}{(1)^2-(\cos\theta)^2}$

$= \frac{(1-\cos\theta)(1-\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}$ $[\because a^2 - b^2 = [(a-b)(a+b)]$

$= \frac{1-\cos\theta}{1+\cos\theta} = R.H.S.$

$\therefore$ L.H.S = R.H.S.

**16.** **Prove that:** $\mathbf{\tan\,48^0\,\tan\,23^0\,\tan\,42^0\,\tan\,67^0 = 1}$. $\left[3\frac{1}{2}\right]$

Ans. $L.H.S.$ = $\tan\,48^0\,\tan\,23^0\,\tan\,42^0\,\tan\,67^0$

$\Rightarrow \tan 48^0 \tan 23^0 \tan(90^0 - 48^0) \tan(90^0 - 23^0)$

$\Rightarrow \tan 48^0 \tan 23^0 \cot 48^0 \cot 23^0$

$\Rightarrow \tan 48^0 \tan 23^0 \cot 48^0 \cot 23^0$

$\Rightarrow \tan 48^0 \tan 23^0 \frac{1}{\tan 48^0} \frac{1}{\tan 23^0} = 1$

Hence, L.H.S = R.H.S.

**17. Find the area of a sector of a circle with radius 6 cm. If angle of the sector is $60^0$.** $\left[3\frac{1}{2}\right]$

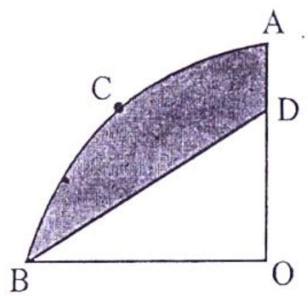
Ans. Radius of a circle (r) = 6 cm

Angle of the sector $(\theta) = 60^0$

Area of the sector $= \frac{\theta}{360} \times \pi r^2$

$= \frac{60}{360} \times \pi \times 6 \times 6 = \frac{132}{7} cm^2.$

**18. OACB is a quadrant of a circle with centre O and radius 3.5 cm. If OD = 2 cm then find the area of quadrant and area of shaded region.** $\left[3\frac{1}{2}\right]$




Ans. The radius of a quadrant OACB 3.5 cm and OD= 2 cm

The area of the quadrant OACB

$= \frac{1}{4} \times \pi r^2$

$= \frac{1}{4} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2 cm^2$

$= \frac{1}{4} \times \frac{22}{7} \times \frac{49}{4} cm^2$

$= \frac{11 \times 7}{8} cm^2$

$= \frac{77}{8} cm^2$

$\Delta OBD$ is a right angled triangle

Here, $OB = \frac{7}{2}\ cm\ and\ OD = 2\ cm$

The area of $\Delta OBD = \frac{1}{2}\ OB \times OD$

$= \frac{1}{2} \times \frac{7}{2} \times 2\ cm^2$

$= \frac{7}{2}\ cm^2$

Hence, the area of the shaded region = The area of quadrant OACB – The area of a $\Delta OBD = \frac{77}{8}\ cm^2 - \frac{7}{2}\ cm^2$

$= \frac{77-28}{8}\ cm^2 = \frac{49}{8}\ cm^2$.

**19. Find the area of a triangle whose vertices are $(-5,-1)$, $(3,-5)$ and $(5,2)$.** $\left[3\frac{1}{2}\right]$

Ans. Let vertices of a given triangle ABC are $A(-5,-1), B(3,-5)\ and\ C\ (5,2)$.

Here, $x_1 = 5\ and\ y_1 = -1$

$x_2 = 3\ and\ y_2 = -5$

$x_3 = 5\ and\ y_3 = 2$

We Know that area of $\Delta ABC = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}[-5(-5-2) + 3(2-(-1)) + 5(-1-(-5))]$

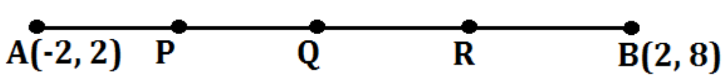
$= \frac{1}{2}[-5 \times -7 + 3 \times 3 + 5 \times 4]$

$= \frac{1}{2}[35 + 9 + 20]$

$= \frac{1}{2} \times 64 = 32$ square unit.

**20. Find the co-ordinates of the points which divides the line segment joining $A(-2,2)$ and B $(2,8)$ into four equal parts.** $\left[3\frac{1}{2}\right]$

Ans. Let the points P, Q and R divides the line segment $A(-2,2)$ and $B\ (2,8)$ into four equal parts.




$\therefore\ AP = PQ = QR = RB$

Now Q is mid point of AB.

By mid-point formula the co-ordinates of the point Q are $\left(\frac{-2+2}{2}, \frac{2+8}{2}\right)$ = (0.5)

R is mid-point of QB.

Then the coordinates of R are $\left(\frac{0+2}{2}, \frac{5+8}{2}\right) = \left(1, \frac{13}{2}\right)$

P is mid-point of AQ.

Then, the coordinates of P are

$\left(\frac{-2+0}{2}, \frac{2+5}{2}\right) = \left(-1, \frac{7}{2}\right)$

Hence, the required points are $P\left(-1, \frac{7}{2}\right)$, $Q(0, 5)$ and R $\left(1, \frac{13}{2}\right)$.

## SECTION-C

**21. An express train takes one hour less than a passenger train to travel 132 m between Mysore and Bangalore. If speed of express train is 11 km/h more than the passenger train then find the average speed of both trains. [5]**

Ans. Let the average speed of the passenger train = $x\ km/h$

And the average speed of the express train = $y\ km/h$

Distance between mysore and Banagalore = 132 km.

Time taken by passenger train to travel a distance 132 km = $\frac{132}{x} hrs.$

Time taken by express train to travel a distance 132 km = $\frac{132}{x+11} hrs.$

According to question, $\frac{132}{x} - \frac{132}{x+11} = 1$

$\Rightarrow 132\left[\frac{1}{x} - \frac{1}{x+11}\right] = 1$

$\Rightarrow 132\left[\frac{x+11-x}{x(x+11)}\right] = 1$

$\Rightarrow \quad \frac{11}{x^2+11x} = \frac{1}{132}$

$\Rightarrow x^2 + 11x = 11 \times 132$

$\Rightarrow x^2 + 11x - 1452 = 0$

which is a quadratic equation in $x$ variable

Here, $a = 1, b = 11, c = -1452$

$D = b^2 - 4ac$

$= (11)^2 - 4 \times 1 \times (-1452)$

$= 121 + 5808 = 5929$

Now, $x = \frac{-b \pm \sqrt{D}}{2a}$

$= \frac{-11 \pm \sqrt{5929}}{2 \times 1}$

$= \frac{-11 \pm 77}{2}$

$= \frac{66}{2}, \frac{-88}{2}$

$= 33, -44$

But speed of the train can't be negative.

So, $x = 33$

Hence, average speed of the passengers train $= 33\ km/h$

And average speed of the express train $= (33 + 1)\ km$

$= 44\ km/h$

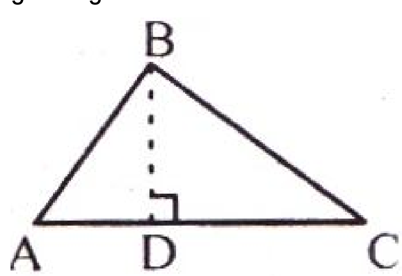
**22.** **In a right triangle, the square of the hypotenuse is kequal to the sum of the squares of the other two sides.** **[5]**

Ans. **Given**

A right angled $\Delta ABC$ which is right angled at B.

**To prove**

$AC^2 = AB^2 + BC^2$




**Construction**

We draw, $BD \perp AC$

**Proof**

$\Delta ADB \sim \Delta ABC$

($\therefore$ Perpendicular is drawn from the vertex of the right angle of right triangle to the hypotenuse then triangle on both side of the perpendicular are similar to whole triangle)

$\Rightarrow \frac{AD}{AB} = \frac{AB}{AC}$ [$\because$ sides are proportional]

Or $AB \times AB = AD \times AC$

Or $AB^2 = AD \times AC$ ............(i)

Also, $\Delta CDB \sim \Delta CBA$

Therefore, $\frac{CD}{BC} = \frac{BC}{CA}$ [∵ sides are proportional]

Or $BC \times BC = CD \times CA$

Or $BC^2 = CD \times CA$ ................(ii)

Now, adding (i) and (ii), we get

$AB^2 = BC^2 = AD \times AC + CD \times AC$

$= AC[AD + CD]$

$= AC \times AC = AC^2.$

**23. As observed from 75 m high light house from the sea level, the angle of depression of two ships are $30^0$ $and$ $45^0$. If one ship is exactly behind the other on the same side of the lighthouse find the distance between the two ships. [5]**

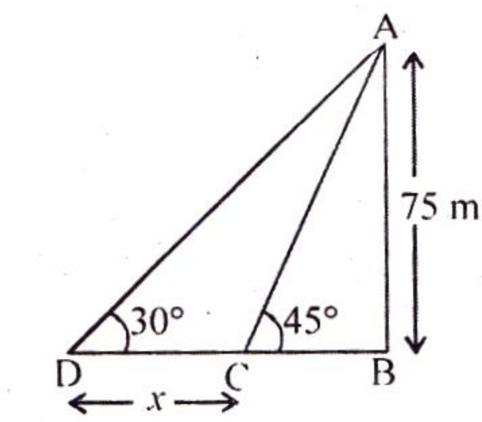
Ans. Let the distance between the ships = $x\ m$

Height of the light house (AB) = 75 m,

D and C are the positions of two ships.

$\angle ADB = 30^0$ (Given)

$\angle ACB = 45^0$ (Given)




In right $\Delta ADB$,

$\tan 30^0 = \frac{AB}{DB}$

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{75}{DB}$

$\Rightarrow \quad DB = (75 \times \sqrt{3})$ ..................(i)

In right $\Delta ABC$,

$\tan 45^0 = \frac{AB}{BC}$

$\Rightarrow \quad 1 = \frac{75}{BC}$

$\Rightarrow \quad BC = 75m$ ....................(ii)

Now the distance between the ships

$x = DB - BC = 75\sqrt{3} - 75 = 75(\sqrt{3} - 1)m.$

**24.** **How many silver coins, 1.75 cm in diameter and thickness of 2 mm must be melted to form a cuboid of dimensions of $5.5\ cm \times 10cm \times 3.5\ cm$? [5]**

Ans. Given

Diameter of the silver coin = 1.75 cm

Radius of the silver coin (r) = $\frac{1.75}{2}cm$

Thickness of the silver coin = 2 mm

= 0.2 cm

Dimensions of cuboid = 5.5 $cm$ × 10$cm$ × 3.5 $cm$

Let the numbers of coin = $n$

We know that

Volume of n coins = Volume of the cuboid

$n \times \pi r^2 h = l \times b \times h$

$\Rightarrow \quad n\frac{22}{7} \times \left(\frac{1.75}{2}\right)^2 \times (0.2) = 5.5 \times 10 \times 3.5$

$\Rightarrow \quad n \times \frac{22}{7} \times \frac{7}{7} \times \frac{7}{8} \times 2 = 55 \times 35$

$\Rightarrow \quad n = \frac{55 \times 35 \times 16}{7 \times 11}$

$\Rightarrow \quad n = 5 \times 5 \times 16 = 400.$

The number of coins = 400.

**25.** **The distribution below gives the weights of 30 students of a class. Find the median weight of the students. [5]**

| **Weight (in kg)** | 40-45 | 45-50 | 50-55 | 55-60 | 60-65 | 65-70 | 70-75 |
|---|---|---|---|---|---|---|---|
| **Number of students** | 2 | 3 | 8 | 6 | 6 | 3 | 2 |

Ans.

| **Weight (in kg)** | **No. of student** $(f_1)$ | **Cumulative frequency** $(c.f.)$ |
|---|---|---|
| 40-45 | 2 | 2 |
| 45-50 | 3 | 2+3=5 |
| 50-55 | 8 | 5+8=13 |
| 55-60 | 6 | 13+6=19 |
| 60-65 | 6 | 19+6=25 |
| 65-70 | 3 | 25+3=28 |
| 70-75 | 2 | 28+2=30 |

Here, we have, n=30

So, $\frac{n}{2} = 15$, therefore the median class is $55 - 60$.

Here $l = 55, n = 30, f = 6, c.f. = 13 \text{ } and \text{ } h = 5$

Using the formula of median,

Median = $l + \left[\frac{\frac{n}{2} - c.f.}{f}\right] \times h$

$$= 55 + \left[\frac{15-13}{6}\right] \times 5 = 55 + \frac{5}{3} = 55 + 1.67 = 56.67 \, kg$$

Hence, median weight of the students = 56.67 kg.