Section - A

Q.1 **Find HCF of 867 and 255 using Euclid's division Algorithm.** $\left[2\frac{1}{2}\right]$

Solution- **Step I :** 867 > 255, we apply the division lemma to 867 and 255, to get

$867 = 255 \times 3 + 102$

Step II : Since the remainder $102 \neq 0$, we apply the division lemma to 255 and 102, to get

$255 = 102 \times 2 + 51$

Step III : Since the remainder $51 \neq 0$, we apply division lemma to 102 and 51, to get

$102 = 51 \times 2 + 0$

Now the remainder is Zero. So our procedure stops. Since the divisor at this stage is 51. So the HCF of 867 and 255 is 51.

Q.2 **Find LCM and HCF of 26 and 91.** $\left[2\frac{1}{2}\right]$

Solution- 26 and 91

We have prime factor of 26

2	26
	13

$26 = 2 \times 13$

We have prime factor of 91

7	91
	13

$91 = 7 \times 13$

HCF (26,91) = Product of the smallest power of each common prime factor in the numbers.

Hence, HCF (26,91) = $13^1 = 13$

LCM (26, 91) = Product of the greatest power of each prime factor involved in the numbers.

Hence, LCM (26, 91) = $2^1 \times 7^1 \times 13^1$

$= 2 \times 7 \times 13$

$= 182$

Verification

LCM × HCF = Product of two numbers

$\Rightarrow 182 \times 13 = 26 \times 91$

$\Rightarrow 2366 = 2366.$

Q.3 **Solve the following pair of linear equation :** $\left[2\frac{1}{2}\right]$

$$x + y = 14$$
$$x - y = 4$$

Solution : Given pair of linear equations

$x + y = 14$ ....(i)

$x - y = 4$ ....(ii)

From equation (i), we get

$y = 14 - x$ ....(iii)

Substituting this value of y in equation

(ii), we get

$x - (14 - x) = 4$

$x - 14 + x = 4$

$\Rightarrow \ 2x = 4 + 14$

$\Rightarrow \ 2x = 18$

$\Rightarrow \ x = \frac{18}{2} = 9$

Substituting the value of x in equation (iii, we get

$y = 14 - 9 = 5$

Hence $x = 9$, $y = 5$.

Q.4 **Find the roots of quadratic equation** $\left[2\frac{1}{2}\right]$

$$100x^2 - 20x + 1 = 0$$

Solution : Let us first split the middle term $-20x\ as - 10x - 10x(because\ 10x \times 10x = 100x^2$

So, $100x^2 - 10x - 10x + 1 = 0$

$\Rightarrow 10x(10x - 1) - 1(10x - 1) = 0$

$\Rightarrow (10x - 1)(10x - 1) = 0$

Either $(10x - 1) = 0\ or\ (10x - 1) = 1$

$\Rightarrow 10x = 1\ or\ 10x - 1 = 0$

$\Rightarrow x = \frac{1}{10}\ or\ x = \frac{1}{10}$

Hence, given equation have two repeated roots, one for each repeated factor.

Q.5 **ABC is an isosceles triangle, right angled at C. Prove that $AB^2 = 2AC^2$** $\left[2\frac{1}{2}\right]$

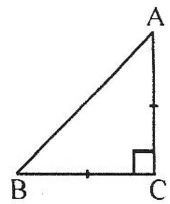
Solution : Given

An isosceles ΔABC which is right angled at C.

$\Rightarrow AC = BC \text{ and } \angle ACB = 90^\circ$

To prove

$AB^2 = 2AC^2$



Proof

In right ΔABC

$AB^2 = AC^2 + BC^2$ $(By\ pythagoras\ \ theorem)$

$AB^2 = AC^2 + AC^2$ $[\because BC = AC]$

$AB^2 = 2AC^2$

$\therefore \quad AB^2 = 2AC^2$

Hence, we proved the result.

Q.6 **If $Q\ (0,1)$ is equidistant from points $P(5,-3)$ and $R\ (x,6)$. Then find the value of x and also find the distance QR and PR.** $\left[2\frac{1}{2}\right]$

Solution : The point Q(0,1) is equidistant from the points P(5,3) and R(x,6

$\therefore QP = QR$

$\Rightarrow \sqrt{(5-0)^2 + (-3-1)^2} = \sqrt{(x-0)^2 + (6-1)^2}$

$\Rightarrow \sqrt{25+16} = \sqrt{x^2+25}$

$\Rightarrow 25 + 16 \quad = x^2 + 25$

$\Rightarrow x^2 = 16$

$\Rightarrow x = \pm 4$

Now, QR = $\sqrt{(x-0)^2 + (6-1)^2}$

$= \sqrt{x^2+25}$

$= \sqrt{16+25}$ $(\because x^2 = 16)$

$= \sqrt{41}$

and PQ = $\sqrt{(x-5)^2 + (6+3)^2}$

When x = 4

PR = $\sqrt{(4-5)^2 + (6+3)^2}$

$= \sqrt{1+81}$

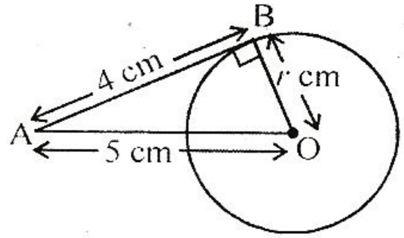
$= \sqrt{82}$

PR = $\sqrt{(-4-5)^2 + (6+3)^2}$

$= \sqrt{81+81} = \sqrt{162} = 9\sqrt{2}$

Q.7 **The length of a tangent from a point A at a distance 5cm from the centre of the circle is 4 cm. Find the radius of the circle.** $\left[2\frac{1}{2}\right]$

Solution :



Let radius of the circle = rcm

The length of the tangent from point A =4cm

The length of point A from the centre of the circle = 5cm

We know, $\angle OBA = 90°$

In right ΔAOB,

$(AO)^2 = (AB)^2 + (OB)^2$

$\Rightarrow (5)^2 = (4)^2 + r^2$

$\Rightarrow r^2 = (5)^2 - (4)^2 = 25 - 16 = 9$

$\Rightarrow r = 3$

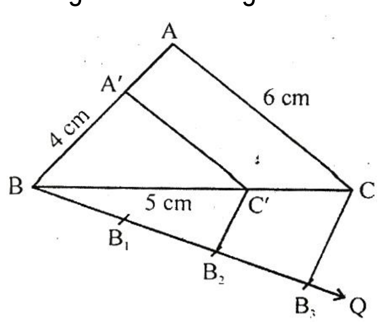
Hence, radius of the circle = 3 cm

Q.8 **Construct a triangle with sides 4 cm, 5 cm & 6 cm and then another triangle similar to it whose sides are $\frac{2}{3}$ of the corresponding sides of first triangle.** $\left[2\frac{1}{2}\right]$

Solution : Given a triangle ABC,of sides 4cm, 5 cm, and 6 cm, we are required to construct a triangle whose sides are $\frac{2}{3}$ of the corresponding sides of ΔABC.

Steps of constructions :

(i) Draw a ray BQ making an acute angle with BC below the side of BC.



(II) Mark 3 points B_1, B_2, B_3 (the greater of 2 and 3 in $\frac{2}{3}$) on BQ, so that $BB_1 = B_1B_2 = B_2B_3$.

(iii) Join C and B_3

(iv) Draw a line through point B_2 (the smaller of 2 and 3 in $\frac{2}{3}$) which is parallel to the line B_3C, to intersect BC at C'.

(v) Through C' draw a line parallel to the line AC to intersect AB at A'.

Hence A $'BC'$ is the triangle, whose sides are $\frac{2}{3}$ of the sides of $\Delta\ ABC$.

Q.9 **A box contains 3 blue, 2 white, 4 red marbles. If a marble is taken out randomly, then calculate the probability that it will be a white marble.** $\left[2\frac{1}{2}\right]$

Solution : Total marbles = 3 + 2 + 4 = 9

Number of white marbles = 2

∴ Probability of getting white marbles = $\frac{2}{9}$

Q.10 **One card is drawn from a well shuffled deck of 52 cards. Find the probability of getting a red face card.** $\left[2\frac{1}{2}\right]$

Solution : Total outcomes = 52

Number of red face card (2King, 2Queen and 2 Jack) = 6

∴ Probability of getting red face card = $\frac{6}{52} = \frac{3}{26}$.

SECTION- B

Q.11 $\boldsymbol{Divide\ (x^4 - 3x^2 + 4x + 5)\ by\ (x^2 + 1 - x).}$ $\left[3\frac{1}{2}\right]$

Solution : $p(x) = (x^4 - 3x^2 + 4x + 5)$,
$g(x) = (x^2 + 1 - x)$
Here, divisor is not in standard from.
The standard from of $g(x) = x^2 - x + 1$
Now, dividend and divisor are in standard from.

$$\begin{array}{r|l} & x^2 + x - 3 \\ x^2 - x + 1 & x^4 + 0x^3 - 3x^2 + 4x + 5 \\ & x^4 - x^3 + x^2 \\ & \underline{- \quad + \quad -} \\ & x^3 - 4x^2 + 4x + 5 \\ & x^3 - x^2 + x \\ & \underline{- \quad + \quad -} \\ & -3x^2 + 3x + 5 \\ & -3x^2 + 3x - 3 \\ & \underline{+ \quad - \quad +} \\ & 8 \end{array}$$

Now, we stop the process because degree of dividend (remainder) becomes less than the degree of divisor.

Hence, remainder = 8

and quotient = $x^2 + x - 3$.

Q.12 **Solve the pair of equations graphically** $\left[3\,\frac{1}{2}\right]$

x - y + 1 = 0

3x + 2y - 12 =0

Solution : Given equations are

x - y + 1 = 0 ....(i)

and 3x + 2y - 12 =0 ....(ii)

Let us draw the graphs of two equations :

We have,

x - y + 1 = 0

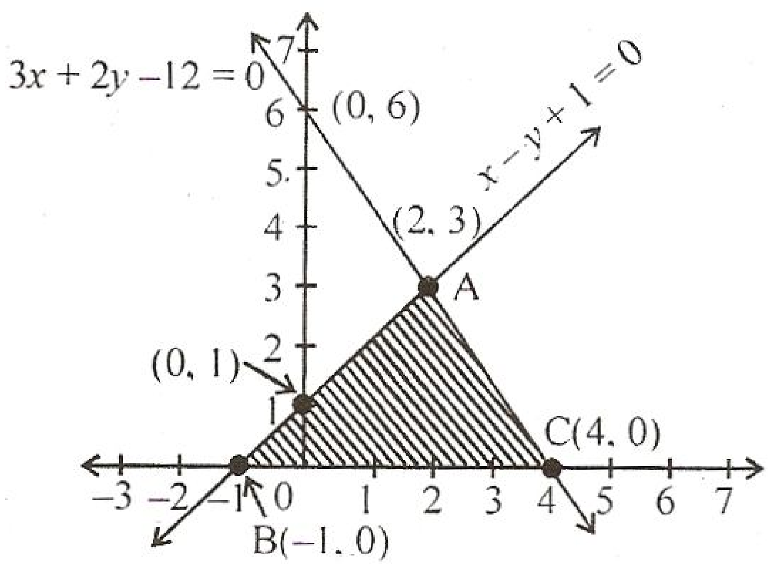
$\Rightarrow \quad y = x + 1$

x	0	-1	2
y	1	0	3

and 3x+2y = 12

$\Rightarrow \quad y = \frac{12-3x}{2}$

x	0	4	2
y	6	0	3



From the graph, we find co-ordinates of the vertices of the triangle ABC formed by these lines and x-axis are : A (2,3) B (-1,0), and C (2,3).

Q.13 **How many three digit numbes are divisible by 7?** $\left[3\frac{1}{2}\right]$

Solution : The given AP which is divisible by 7 is 105, 112, 119, ……… 994

Let the three digit number, which is divisible by 7 are n.

Here a = 105, d = 7 and a_n = 994

We know that

$$a_n = [a + (n-1)d]$$

$\Rightarrow 994 = [105 + (n-1)7]$

$\Rightarrow 994 = 105 + 7n - 7$

$\Rightarrow 7n + 98 = 994$

$\Rightarrow 7n = 994 - 98$

$\Rightarrow 7n = 896$

$\Rightarrow n = \frac{896}{7} = 128$

Hence, the three digit numbers which is divisible by 7 are 128.

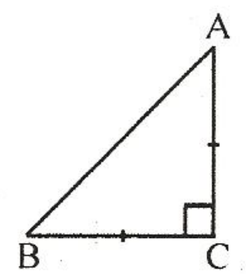
Q.14 **Triangle ABC is an isosceles triangle with AC = BC.**

If $AB^2 = 2AC^2$ then prove that triangle ABC is a right angled triangle. $\left[3\frac{1}{2}\right]$

Solution : **Given**

An isosceles ΔABC which $AC = BC$

$AB^2 = 2AC^2$



To Prove

ΔABC is a right triangle.

Proof

In right $\Delta ABC, AC = BC$ Given ....(i)

$AB^2 = 2AC^2$ (Given) ....(ii)

Now, $AC^2 + BC^2 = AC^2 + AC^2$ (using (i))

$\Rightarrow AC^2 + BC^2 = 2AC^2$

$\Rightarrow AC^2 + BC^2 = AB^2$ (using (ii))

Hence, by the converse of the pythagoras theorem, we have ΔABC right angle at C.

Q.15 **Prove that :** $\frac{\sin\theta - \sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$ $\left[3\frac{1}{2}\right]$

Solution : L.H.S.=$\frac{\sin\theta - \sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$

$= \frac{\sin\theta \times (1 - 2\sin^2\theta)}{\cos\theta \times (2\cos^2\theta - 1)}$

$= \frac{\sin\theta \times (\cos^2\theta + \sin^2\theta - 2\sin^2\theta)}{\cos\theta \times [2\cos^2\theta - (\cos^2\theta + \sin^2\theta)]}$

$(\because \cos^2\theta + \sin^2\theta = 1)$

$= \frac{\sin\theta \times (\cos^2\theta - \sin^2\theta)}{\cos\theta \times (\cos^2\theta - \sin^2\theta)}$

$= \frac{\sin\theta}{\cos\theta} = \tan\theta = R.H.S$

$\therefore L.H.S. = R.H.S.$

Q.16 **Evaluate :** $2\tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$ $\left[3\frac{1}{2}\right]$

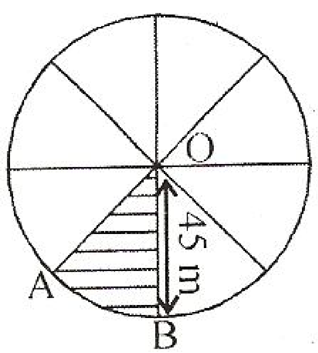
Solution : $2\tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$

$= 2(1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$

$= 2 + \frac{3}{4} - \frac{3}{4} = 2.$

Q.17 **An umbrella has 8 ribs which are equally spaced. Assuming umbrella to be a flat circle of radius 45 cm. Find the area between the two consecutive ribs of the umbrella.** $\left[3\frac{1}{2}\right]$

Solution :



Radius of circle = 45 cm

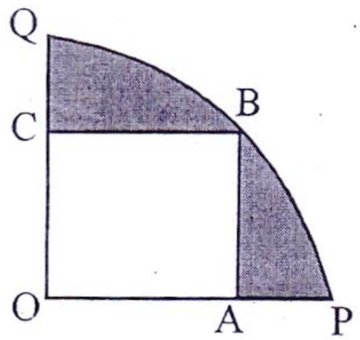
Angle subtended at the centre between two consecutive ribs = $\frac{360^\circ}{8} = 45^\circ$

Therefore, area between two consecutive ribs of the umbrella = The area of the shaded sector OAB = $\frac{\theta}{360} \times \pi r^2$

$= \frac{45^\circ}{360} \times 3.14 \times 45 \times 45$

$= \frac{22275}{28}\, cm^2.$

Q.18 **In figure, a square OABC is inscribed in a quadrant OPBQ. If OA = 20 cm. Find Area of the shaded region.** $(\pi = 3.14)$ $\left[3\frac{1}{2}\right]$



Solution : Let draw a diagonal OB in square OABC

AB = BC = OA = OC = 20 cm

Because OABC is a square, so $\angle QOP = 90^\circ$

Diagonal (OB) = $\sqrt{OA^2 + OB^2}$

OB $= \sqrt{20^2 + 20^2}$

$= \sqrt{400 + 400}$

$= 20\sqrt{2}$ cm

But in figure, we have

Diagonal = radius of the circle = $20\sqrt{2}$ cm.

Therefore, the area of shaded portion = Area of quadrant OPQ of circle - Area of square

$=\left[\frac{\theta}{360} \times \pi r^2 - (side)^2\right]$

$= \left[\frac{90}{360} \times \frac{22}{7}\left(20\sqrt{2}\right)^2 - (20)^2\right]$

$=\left[\frac{1}{4} \times 3.14 \times 800 - 400\right]$

= 628 - 400 = 228 cm^2.

Q.19 **Find the area of triangle whose vertices are (2, 3), (-1,0), (2,-4).** $\left[3\frac{1}{2}\right]$

Solution : Let vertices of a given triangle ABC are A (2,3), B (-1,0) and C (2,-4)

Hence, $x_1 = 2 \; and \; y_1 = 3$

$x_2 = -1 \; and \; y_2 = 0$

$x_3 = 2 \; and \; y_3 = -4$

We know that area of ΔABC

$=\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

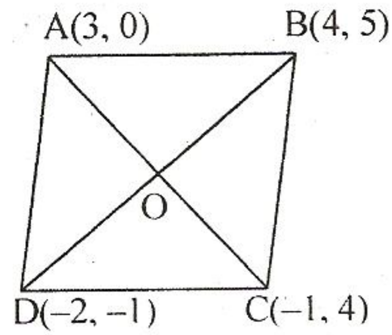
$=\frac{1}{2}[2(0-4) - 1(-4-3) + 2(3-0)]$

$=\frac{1}{2}(8 + 7 + 6) = \frac{1}{2} \times 21$

= 10.5 sq. units.

Q.20 **Find the area of a rhombus if its vertices are (3, 0), (4, 5), (-1, 4) and (-2, -1) taken in order.** $\left[3\frac{1}{2}\right]$

Solution : Let A (3, 0), B (4, 5), C (-1, 4) and D (-2, -1) are the vertices of the rhombus.



Here, diagonal BD

$= \sqrt{[4-(-2)]^2 + (5-(-1))^2}$

$= \sqrt{(4+2)^2 + (5+1)^2} = \sqrt{(6)^2 + (6)^2}$

$= \sqrt{36+36}$

BD $= \sqrt{72} = 6\sqrt{2}.$

and diagonal AC $= \sqrt{(-1-3)^2 + (4-0)^2}$

$= \sqrt{(-4)^2 + (4)^2}$

$= \sqrt{16+16}$

AC $= \sqrt{32} = 4\sqrt{2}$

We know that area of rhombus

$= \frac{1}{2}(BD \times AC) = \frac{1}{2}(6\sqrt{2} \times 4\sqrt{2})$

= 24 square units.

SECTION- C

Q.21 **The difference of square of two numbers is 180. If square of smaller number is equal to 8 times the bigger number then find the two numbers.** **[5]**

Solution : Let smaller number = x

and larger number = y

According to first condition,

$x^2 = 8y$ ....(i)

According to second condition,

$y^2 - x^2 = 180$ ......(ii)

Putting the value of x^2 from equation, (i) in equation (ii), we get

$y^2 - 8y - 180 = 0$ ....(iii)

Which is a quadratic equation.

$\Rightarrow y^2 - 18y + 10y - 180 = 0$

$\Rightarrow y(y - 18) + 10(y - 18) = 0$

$\Rightarrow (y - 18)(y + 10) = 0$

Either $y - 18 = 0$ or $y + 10 = 0$

$\Rightarrow y = 18 \; or \; y = -10$

when y = 18 then $x^2 = 8y$

$\Rightarrow x^2 = 8 \times 18$

$\Rightarrow x^2 = 144$

$\Rightarrow x = \sqrt{144}$

$\Rightarrow x = \pm 12$

when y = - 10 then $x^2 = 8y$

$\Rightarrow x^2 = 8 \times -10$

$\Rightarrow x^2 = -80$

$\Rightarrow x = \sqrt{-80}$

$\Rightarrow x \notin R$

Hence, Y = 18

and x = 12 or -12

Case- 1

If smaller number = 12

then larger number (y) = $\frac{x^2}{8} = \frac{(12)^2}{8} = \frac{144}{8} = 18$

Case -2

If smaller number =-12

then larger number (y)

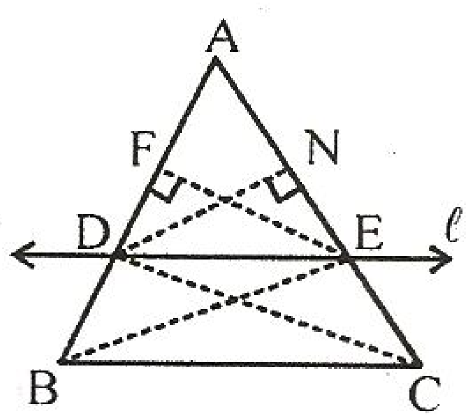
$= \frac{x^2}{8} = \frac{(-12)^2}{8}$

$= \frac{144}{8} = 18.$

Q.22 **If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct point, the other two sides are divided in the same ratio.** **[5]**

Solution : **Given**

ΔABC and a line parallel to BC intersects Ab at D and AC at E



To prove

$\frac{AD}{DB} = \frac{AE}{EC}$

Construction

We join B and E, C and D then draw EF $\perp AB, and\ DN \perp AC$

Proof

We have, $ar\ (\Delta BDE) = ar(\Delta CDE)$

(The two areas are equal because the two triangles are on the same base DE and between same parallel line DE and BC)

We have

$\frac{ar\ (\Delta ADE)}{ar\ (\Delta BDE)} = \frac{\frac{1}{2} \times AD \times EF}{\frac{1}{2} \times BD \times EF} = \frac{AD}{BD}$

$ar(\Delta ADE) = \frac{AD}{BD} \times ar(\Delta BDE) \quad \dots..(i)$

Similarly $\frac{ar\ (\Delta ADE)}{ar(\Delta CDE)} = \frac{\frac{1}{2} \times AE \times DN}{\frac{1}{2} \times EC \times DN}$

$= \frac{AE}{EC}$

$ar(\Delta ADE) = \frac{AE}{EC} \times ar\ (\Delta CDE)$ .....(ii)

From (i) and (ii), we get

$\frac{AD}{BD} \times ar(\Delta BDE) = \frac{AE}{EC} \times ar(\Delta CDE)$ $[\because ar\ (\Delta BDE) = ar\ (\Delta CDE)]$

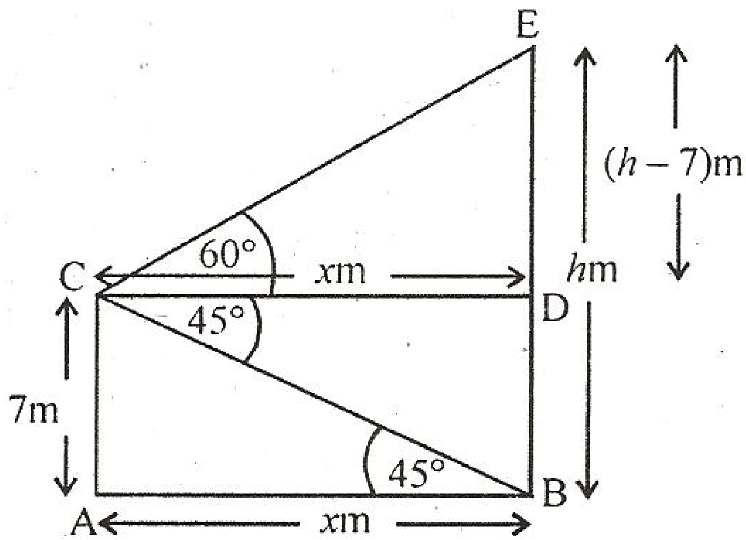
$\Rightarrow \frac{AD}{BD} = \frac{AE}{EC}$

Hence, we proved the result.

Q.23 From the top of a 7m high Bulding the angle of elevation of the top of a cable tower is 60^0 and the angle of depression of its foot is 45^0. Determine the height of the tower. [5]

Solution Let the height of the tower = hm

and let the distance between foot of the tower and foot of the building = xm



From the top of a building, the angle of elevaion of the top of a cable tower = 60^0

$\Rightarrow \angle ECD = 60^0$

From the top of a building the angle of depression of = 45^0

$\Rightarrow \angle CBA = 45^0$

(Alternate, angles $\angle DCB = \angle CBA$)

In right ΔCBA,

$\frac{CA}{AB} = \tan 45^0 \Rightarrow \frac{7}{x} = 1 \Rightarrow x = 7m$

Now, right ΔCBA,

$\frac{ED}{CD} = \tan 60^0$

$\Longrightarrow \frac{EB-DB}{x} = \sqrt{3}$

$\Longrightarrow \frac{h-7}{7} = \sqrt{3}\ (\because x = 7m)$

$\Rightarrow h = 7\sqrt{3} + 7 \Rightarrow h = 7(\sqrt{3} + 1)$

Hence, height of the cable tower = $7(\sqrt{3} + 1)m$.

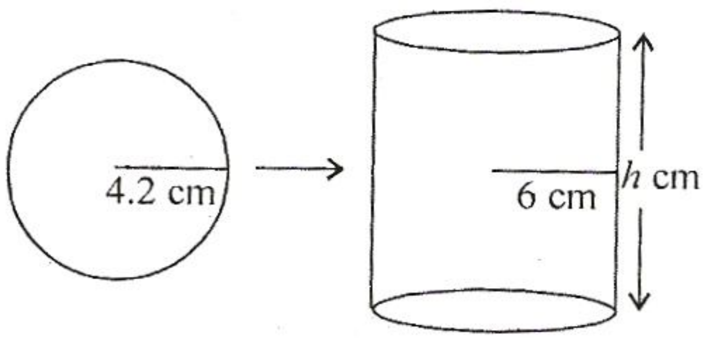
Q.24 A metallic sphere of radius 4.2cm is melted and recast into the shape of cylinder of radius 6 cm. Find the height of the cylinder. [5]

Ans. Given

Radius of the sphere $(r_1) = 4.2\ cm$

Radius of the cylinder $(r_2) = 6\ cm$

LEt the height of the cylinder $= h\ cm$



We know that

Volume of sphere = Volume of cylinder

$\Rightarrow \frac{4}{3}\pi r_1^3 = \pi r_2^2 h$

$\Rightarrow h = \frac{4}{3}\pi r_1^3 \times \frac{1}{\pi r_2^2}$

$= \frac{4}{3} . \frac{(4.2)\times(4.2)\times(4.2)}{6\times 6} = 2.74\ cm$

Q.25 The folloiwng table givens the distribution of life time of 400 neon Lamps. Find the median life time of lamp. [5]

Life time (in hours)	No. of Lamps
1500-2000	14
2000-2500	56
2500-3000	60
3000-3500	86
3500-4000	74
4000-4500	62
4500-5000	48

ANS.

Life time (in hours)	Number of lamps (f_1)	Cumulative Frequency (c.f.)
1500-2000	14	14
2000-2500	56	70
2500-3000	60	130
3000-3500	86	216
3500-4000	74	290
4000-4500	62	352
4500-5000	48	400
	n=200	

Here, we have $n = 200$, so $\frac{n}{2} = 100$

Hence, medium class is 3000 – 3500

Therefore, $l = 3000, f = 86, c.f. = 130$ and $h = 500$

Using the formula of median,

Median = $l + \left[\frac{\frac{n}{2} - c.f.}{f}\right] \times h$

$= 3000 \left[\frac{200-130}{86}\right] \times 500$

$= 3000 + 406.98$

$= 3406.98$ hours

Hence, median life time of a lamp = 3406.98 hours