SECTION-A

1. Find HCF of 135 and 225 using Euclid's division Algoritum. $\left[2\frac{1}{2}\right]$

Ans. **Step I:** Since 225>135, we apply the division lemma, to get 225 = 135× 1 + 90.

Step II: Since the remainder 90 ≠ 0, we apply the division lemma to 135 and 90, to get 135= 90 × 1 + 45.

Step III: Since the remainder 45≠ 0, so we again apply the division lemma to 90 and 45, to get 90 = 45 × 2 + 0.

The remainder has now became zero, so our procedure stops.

Since the divisor at this stage is 45, So the HCF of 225 and 335 is 45.

2. Find LCM and HCF of 510 and 92. $\left[2\frac{1}{2}\right]$

Ans. We have prime factor of 510

2	510
3	255
5	85
	17

= 2 × 3 × 5 × 17

Also, prime factor of 92

2	92
2	46
	23

= 2 × 2 × 23 = 2^2 × 23.

HCF (510, 92) = Product of the smallest power of each common factor in the numbers.

Hence, HCF (510, 92)=2.

LCM (510, 92) = product of the greatest power of each prime factor involved in the numbers.

Hence, LCM (510, 92) = 2^2 × 3 × 5 × 17 × 23 = 23460

Verfication:

$LCM \times HCF$ = Product of numbers

$\Rightarrow 23460 \times 2 = 510 \times 92$

$\Rightarrow 46920 = 46920.$

3. Solve the following pair of Linear equation: $\left[2\frac{1}{2}\right]$

$$\boldsymbol{x + y = 5}$$

$$\boldsymbol{2x - 3y = 4}$$

Ans. Given pair of linear equations

$x + y = 5$ …………(i)

$2x - 3y = 4$ ………….(ii)

From equation (i), we get

$x = 5 - y$ …………….(iii)

Substitution the value of x in equation (ii), we get

$2(5 - y) - 3y = 4$

$\Rightarrow \quad 10 - 2y - 3y = 4$

$\Rightarrow \quad 10 - 5y = 4$

$\Rightarrow \quad 5y = 4 - 10$

$\Rightarrow \quad 5y = -6$

$\Rightarrow \quad y = \frac{-6}{-5} = \frac{6}{5}$

Substituting the value of y in equation (iii), we get

$x = 5 - \frac{6}{5} = \frac{25-6}{5} = \frac{19}{5}$

Hence, $x = \frac{19}{5}, y = \frac{6}{5}$.

4. Find the roots of quadratic equation $2x^2 - x + \frac{1}{8} = 0$. $\left[2\frac{1}{2}\right]$

Ans. $\Rightarrow \quad 16x^2 - 8x + 1 = 0$

Let us first split the middle term $-8x$ as $-4x - 4x$. [because$(-4x) \times (-4x) = 16x^2$]

So, $16x^2 - 4x - 4x + 1 = 0$

$\Rightarrow \quad 4x(4x - 1) - 1(4x - 1) = 0$

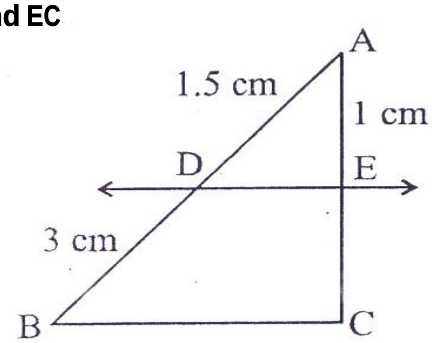
$(4x - 1)(4x - 1) = 0$

Either $4x - 1 = 0$ or $4x - 1 = 0$

$\Rightarrow x = \frac{1}{4}$ or $x = \frac{1}{4}$

Hence, given equation have two repeated roots, one for each repeated factor.

5. **In figure, $DE||BC$. Find EC** $\left[2\frac{1}{2}\right]$



Ans. In fig. (i), we have

AD = 1.5 cm

DB = 3cm and AE = 1 cm

DE || BC (Given)

$\frac{AD}{DB} = \frac{AE}{EC}$ (By Basic proportionality Theorem)

$\Rightarrow \frac{1.5}{3} = \frac{1}{EC}$

$\Rightarrow EC = \frac{3}{1.5} = 2\ cm$

6. **If point (x, y) is equidistant from two points (3, 6) and (-3, 4), then find relation between x and y.** $\left[2\frac{1}{2}\right]$

Ans. Three points $P(x, y), A(3, 6) and\ B(-3, 4)$ are given points such that PA= PB

$\sqrt{(x-3)^2 + (y-6)^2} = \sqrt{(x-3)^2 + (y-4)^2}$

$\Rightarrow (x-3)^2 + (y-6)^2 = (x-3)^2 + (y-4)^2$

$\Rightarrow (x^2 - 6x + 9) + (y^2 - 12y + 36) = (x^2 + 6x + 9) + (y^2 - 8y + 16)$

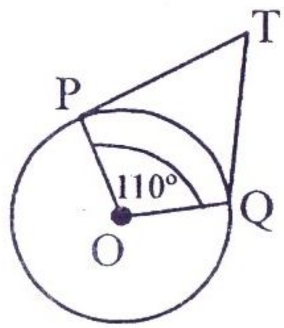
$\Rightarrow -6x - 12y + 45 = 6x - 8y + 25$

$\Rightarrow 12x + 4y - 20 = 0$

$\Rightarrow 3x + y - 5 = 0$

Which is the required relation.

7. If TP and TQ are two tangent lines of a circle with centre O and $\angle POQ = 110^0$ then find $\angle PTQ$. $\left[2\frac{1}{2}\right]$



Ans. From the figure, we have

$\angle POQ$ = 110^0 and $\angle OPT$ = $\angle OQT$ = 90^0

Therefore, $\angle POQ$ + $\angle PTQ$ = 180^0

$\Rightarrow$ 110^0 + $\angle PTQ$ = 180^0

$\Rightarrow$ $\angle PTQ$ = 180^0 – 110^0 = 70^0

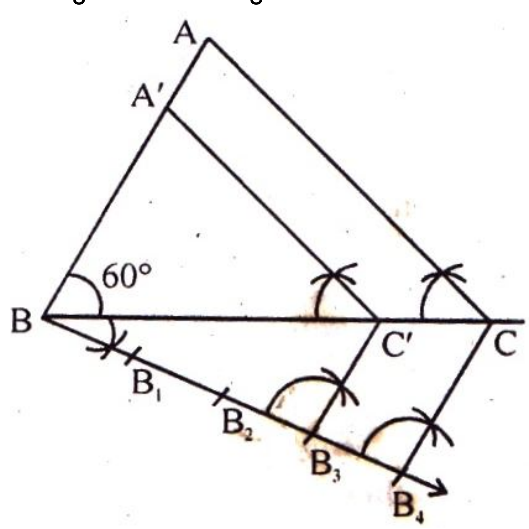
Hence, $\angle PTQ$ = 70^0.

8. Draw a triangle ABC in which BC = 6cm, AB = 5 cm, $\angle B = 60^0$. Then construct another triangle whose sides are $\frac{3}{4}$ of the corresponding sides of triangle ABC. $\left[2\frac{1}{2}\right]$

Ans. Given a triangle ABc with side BC = 6cm, AB = 5cm and $\angle ABC$ = 60^0, we are required to construct a triangle whose sides are $\frac{3}{4}$ of the corresponding sides of the triangle ABC.

Steps of constructions:

(i) Draw a ray BQ making an acute angle with BC below the side of BC.



(ii) Mark 4 points (the greater of 3 and 4 in $\frac{3}{4}$) B_1, B_2, B_3 *and* B_4 on BQ, so that $BB_1 = B_1B_2 = B_2B_3 = B_3 B_4$.

(iii) Join B_4 to C.

(iv) Draw a line through B_3(the smaller of 3 and 4 in $\frac{3}{4}$) parallel to the line B_4C to intersect BC at C'.

(v) Draw a line through point C' parallel to the line AC to intersect BA at A'.

Hence A'BC' is a triangle whose sides are $\frac{3}{4}$ of the corresponding sides of ΔABC.

Justification

$\Delta ABC - A'BC'$ (By AA similarity criteria)

Therefore, $\frac{BC'}{BC} = \frac{BA'}{BA} = \frac{A'C''}{AC}$

But $\frac{BC'}{BC} = \frac{3}{4} \left(\frac{BB_3}{BB_4} = \frac{BC'}{BC} = \frac{3}{4}\right)$

So, $\frac{BC'}{BC} = \frac{BA'}{BA} = \frac{A'C''}{AC} = \frac{3}{4}$

9. A Box contains 5 red, 8 white and 4 green marbles. One marble is taken out of the box randomly. Find the probability that the marble taken out will be red. $\left[2\frac{1}{2}\right]$

Ans. Numbers of red marbles = 5

Number of white marbles = 8

Number of green marbles = 4

Total number of marbles = 5+8+4=17

∴ Probability of getting red marble = $\frac{5}{17}$

10. A dice is thrown once. Find the probability of getting a prime number. $\left[2\frac{1}{2}\right]$

Ans. Coming possibilities = (1, 2, 3, 4, 5, 6) = 6

∴ Probability of getting prime number = $\frac{3}{6} = \frac{1}{2}$

SECTION-B

11. Divide: $(x^4 - 5x + 6)$ by $(2 - x^2)$. $\left[3\frac{1}{2}\right]$

Ans. First of all we write dividend and divisior in the standard form.

$p(x) = x^4 - 5x + 6, g(x) = -x^2 + 2$

$$
\begin{array}{r|l}
 & -x^2 - 2 \\
-x^2 + 2 & x^4 - 5x + 6 \\
 & x^4 \qquad\quad - 2x^2 \\
 & - \qquad\qquad + \\
\hline
 & 2x^2 - 5x + 6 \\
 & 2x^2 \qquad - 4 \\
 & - \qquad\quad + \\
\hline
 & -5x + 10
\end{array}
$$

Now, we stop the process because degree of dividend (remainder) becomes less thean divisor.

Hence, quotient = $(-x^2 + 2)$

and remainder = $-5x + 10$.

12. Solve the pair of equation graphically. $\left[3\frac{1}{2}\right]$

$$2x + y - 6 = 0$$

$$4x - 2y - 4 = 0$$

Ans. Graphically representation

We have, $2x + y - 6 = 0$

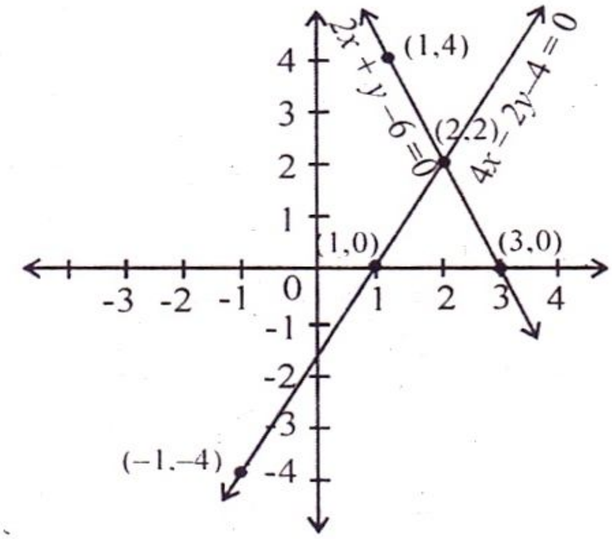
$\Rightarrow \frac{-y+6}{2}$

x	3	2	1
y	0	2	4

and $4x - 2y - 4 = 0$

$\Rightarrow x = \frac{4+2y}{4}$

x	1	2	-1
y	0	2	-4



From the graph, we find two intersecting lines. Therefore, the given equations has unique solution.

Hence $x = 2, y = 2$

13. Find the sum of first 22 terms of an AP in which d=7 and 22nd term is 149. $\left[3\frac{1}{2}\right]$

Ans. **Given**

$n = 22, d = 7$ and $a_{22} = 149$

We know that

$a_n = a + (n-1)d$

$\Rightarrow$ $a_{22} = a + (22-1)d$ $(\because n = 22)$

$\Rightarrow$ $149 = a + (22-1) \times 7$

$\Rightarrow$ $149 = a + 21 \times 7$

$\Rightarrow$ $149 = a + 147$

$\Rightarrow$ $a = 149 - 147 = 2$

We know that

$S_n = \frac{n}{2}[2a + (n-1)d]$

So, $S_{22} = \frac{22}{2}[2(2) + (22-1)7]$ $(\because n = 22)$

$= 11[4 + 21 \times 7]$

$= 11 \times [4 + 147]$

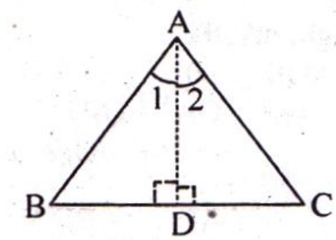
$= 11 \times 151 = 1661$

Hence, sum of first 22 terms is 1661.

14. **ABC is an equilateral triangle of side $2a$. Find each of its altitudes.** $\left[3\frac{1}{2}\right]$

Ans. **Given**

An equilateral triangle ABC of each side $2a$.



Construction

$AD \perp BC$

In ΔADB and ΔADC,

$AD = AD$ $(Common\ side)$

$\angle ABD = \angle ACD$ (opposite anles of equal side)

$\angle ADB = \angle ADC$ $(Each\ 90^0)$

Hence, $\Delta ADB \cong \Delta ADC$ $(ASA\ congruency)$

$\Rightarrow \quad BD = CD = \frac{1}{2}BC = a$

Now, from right ΔABD by Pythagoras

Therorem we get

$$AB^2 = AD^2 + BD^2$$

$\Rightarrow \qquad (2a)^2 = AD^2 + a^2$

$\Rightarrow \quad AD^2 = 4a^2 - a^2 = 3a^2 \Rightarrow \quad AD = \sqrt{3}a$

Hence, altitude of an equilateral $\Delta ABC = \sqrt{3}a$.

15. **Prove that:** $\frac{\mathbf{1+\sec A}}{\mathbf{\sec A}} = \frac{\boldsymbol{sin^2 A}}{\mathbf{1-\cos A}}$. $\left[3\frac{1}{2}\right]$

Ans. L.H.S. $= \frac{1+\sec A}{\sec A}$

$$= \frac{1+\frac{1}{\cos A}}{\frac{1}{\cos A}} = 1 + \cos A$$

$$= \frac{(1+\cos A)\times(1-\cos A)}{1-\cos A}$$

$= \frac{(1)^2-(\cos A)^2}{1-\cos A}$

$= \frac{1-cos^2 A}{1-\cos A} = \frac{sin^2 A}{1-\cos A} = R.H.S.$

$\therefore \quad L.H.S. = R.H.S.$

16. If $\tan A = \cot B$ $then\ prove\ A + B = 90^0$. $\left[3\frac{1}{2}\right]$

Ans. We have

$\tan A = \cot B$

$\Rightarrow \quad \tan A = \tan(90^0 - B) \qquad (\because \tan(90 - \theta) = \cot\theta)$

$\Rightarrow \quad A = 90^0 - B$

$\Rightarrow \quad A + B = 90^0$

Hence, we proved the result.

17. The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 mintues. $\left[3\frac{1}{2}\right]$

Ans. Radius = length of minute hand = 14 cm

The angle covered by minute hand in 60 minutes = 360^0

The angle covered by minute hand in one minute = $\frac{360}{60} = 6^0$.

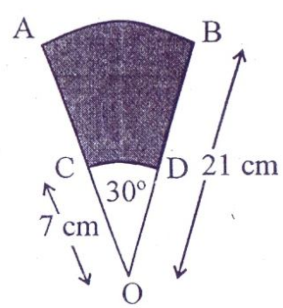
Then angle covered by minute hand in five minutes = $6^0 \times 5 = 30^0$

Therefore, the area swept by the minute hand in 5 minutes = $\frac{\theta}{360} \times \pi r^2$

$= \frac{30^0}{360} \times \frac{22}{7} \times 14 \times 14$

$= \frac{154}{3}\ cm^2$.

18. AB and CD are respectively arcs of two concentric circles of radii 21 cm and 7 cm and centre O. If $\angle AOB = 30^0$. Find the area of shaded region. $\left[3\frac{1}{2}\right]$



Ans. Radius of smaller arc $CD\ (r_1) = 7\ cm$

Radius of bigger arc AB $(r_2) = 21\ cm$

Therefore, the area of shaded portion = The area of sector OAB – the area of sector OCD

$= \left(\frac{\theta}{360}\pi r_2^2 - \frac{\theta}{360}\pi r_1^2\right)$

$= \frac{\theta}{360} \times \pi(r_2^2 - r_1^2)$

$= \frac{\theta}{360} \times \pi[(21)^2 - (7)^2]$

$= \frac{30^0}{360} \times \frac{22}{7} \times [441 - 49]$

$= \frac{1}{12} \times \frac{22}{7} \times 392 = \frac{308}{3}\ cm^2$

Hence, the area of shaded region = $\frac{308}{3}\ cm^2$

19. Find the value of k if the three points $(7, -2), (5, 1)\ and\ (3, k)$ are collinear. $\left[3\frac{1}{2}\right]$

Ans. Let $A(7, -2), B(5, 1), C(3, k)$ are the given points.

$x_1 = 7, x_2 = 5, x_3 = 3$

$y_1 = -2, y_2 = 1, y = k$

If the points are collinear then the condition is

$= [x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = 0$

$\Rightarrow \frac{1}{2}\{7(1 - k) + 5(k—2) + 3(-2 - 1) = 0$

$\Rightarrow \frac{1}{2}(7 - 7k + 5k + 10 - 6 - 3) = 0$

$\Rightarrow \frac{1}{2}(8 - 2k) = 0$

$\Rightarrow 8 - 2k = 0$

$\Rightarrow -2k = -8$

$\Rightarrow k = \frac{-8}{-2} = 4$

Hence, k = 4

20. Find the co-ordinates of point A where AB is diameter of circle whose centre is $(2, -3)$ and co-ordinates of points B are $(1, 4)$. $\left[3\frac{1}{2}\right]$

Ans. Let the coordinates of the point A be (x, y).

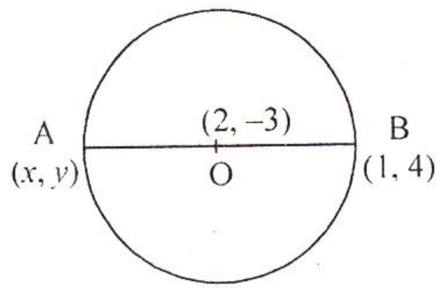
We know that centre of circle is the mid point of the diameter.

Now $O(2, -3)$ is the mid-point of the diameter AB.

Here, the coordinates of A are (x, y) and B are $(1, 4)$.

$\Rightarrow \frac{x+1}{2} = 2$

$\Rightarrow x + 1 = 4 \Rightarrow x = 3$



and $\frac{y+4}{2} = -3$

$\Rightarrow y + 4 = -6$

$\Rightarrow y = -10$

$\Rightarrow x = 3 \; and \; y = -10$

Hence, the coordinates of A is (3, −10).

SECTION-C

21. Sum of the areas of two squares is $468\, m^2$. If difference of their perimeters is 24 m then find the sides of the two squares. [5]

Ans. Let side of first square = x metre

Side of seond square = y metre

So, area of first square = $x^2 m^2$

Area of second square = $y^2 m^2$

Therefore, perimetre of first square = $4x$ metre

Perimetre of second square = 4y metre

According to first condition,

$x^2 + y^2 = 468$ ....(i)

According to second condition,

$4x - 4y = 24$

$\Rightarrow \quad x - y = 6$

$x = y + 6$ ……………(ii)

Putting the value of x in (i), we get

$(y + 6)^2 + y^2 = 468$

$\Rightarrow \quad y^2 + 36 + 12y + y^2 = 468$

$\Rightarrow \quad 2y^2 + 12y + 36 - 468 = 0$

$\Rightarrow \quad 2y^2 + 12y - 432 = 0$

$\Rightarrow \quad y^2 + 6y - 216 = 0$ …………(iii)

Equation (iii) is a quadratic equation,

Here $a = 1$, $b = 6$, $c = -216$

$D = b^2 - 4ac$

$= 36 - 4 \times 1 \times -216$

$= 36 + 864 = 900$

By using quadratic formula,

$y = \frac{-b \pm \sqrt{D}}{2a}$

$= \frac{-6 \pm \sqrt{900}}{2}$

$= \frac{-6 \pm 30}{2} = \frac{24}{2}, \frac{-36}{2}$

$y = 12, -18$

Because side can not be negative,

So, $y = 12$.

Put the value of y in equation (ii), we get $x = 6 + 12 = 18$

So, side of first square = 18 metre

and side of second square = 12 metre.

22. In a triangle, if square of one side is equal to the sum of the square of the other two sides, then the angle opposite the first side is a right angle. Prove it. [5]

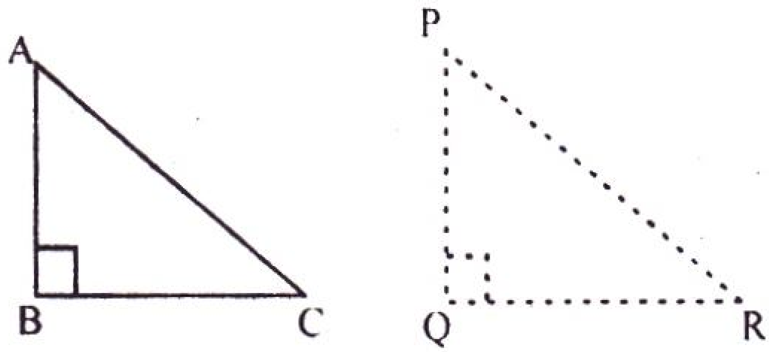
Ans. **Given**

A ΔABC such that $AB^2 + BC^2 = AC^2$

To prove

$\angle B = 90^0$

Consruction



We construct a ΔPQR right angled at Q such that PQ = AB and QR = BC

Proof

In right ΔPQR, we have

$PR^2 = PQ^2 + QR^2$ (By Pythagoras Theorem)

Or $PR^2 = AB^2 + BC^2$ ………………(i)

$(\because PQ = AB \text{ and } QR = BC)$

But $AC^2 = AB^2 + BC^2$ (Given) ………………..(ii)

From (i) and (ii), we get

$PR^2 = AC^2$

$\Rightarrow \quad PR = AC \text{ or } \Delta ABC \cong \Delta PQR$

(SSS congruency)

$\Rightarrow \angle B = \angle Q = 90^0$

(Corresponding angles of similar triangles are equal)

Hence, $\angle B = 90^0$

Hence, we proved the result. This is the converse of the Pythagoras Theorem.

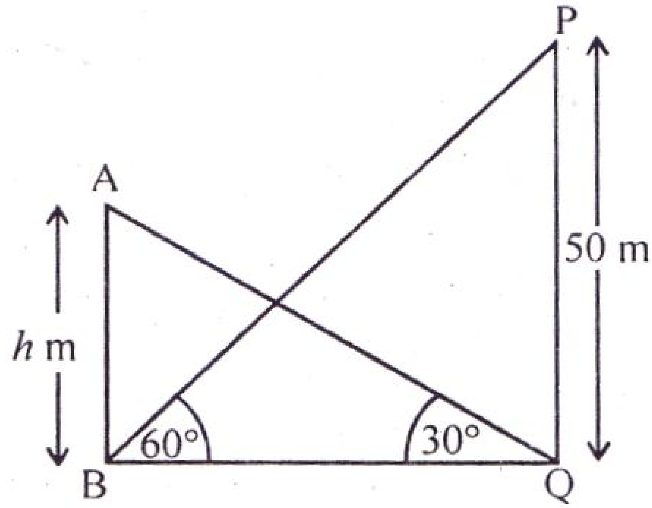
23. The angle of elevation of the top of a building from the foot of the tower is 30^0 and the angle of elevation of the top of tower from foot of building is 60^0. If the tower is 50 m high then find the height of the building. [5]

Ans. Let height of the building = h m

Height of the tower = 50 m

The angle of elevation of the top of a building from the foot of the tower = 30^0

The angle of elevation of the top of the tower from the foot of building = 60^0



In right ΔAQB,

$$\frac{h}{BQ} = \tan 30^0$$

$$\Rightarrow \frac{h}{BQ} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow BQ = h\sqrt{3} \quad \text{.......(i)}$$

In right ΔPBQ

$$\frac{50}{BQ} = \tan 60^0$$

$$\Rightarrow \frac{50}{BQ} = \sqrt{3}$$

$$\Rightarrow BQ = \frac{50}{\sqrt{3}} \quad \text{.......(ii)}$$

Form equation (i)and (ii), we get

$$h\sqrt{3} = \frac{50}{\sqrt{3}}$$

$$\Rightarrow h = \frac{50}{3}$$

$$\Rightarrow h = 16\frac{2}{3}\ m$$

Hence, the height of tower = = $16\frac{2}{3}\ m$.

24. A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm. Which is surmounted by another cylinder of height 60 cm and base radius 8 cm. Find the mass of the pole given that $1\ cm^3 = 8\ gm$ ($\pi = 3.14$) [5]

Ans. Given

Height of larger cylinder $(h_1) = 220\ cm$

Height of smaller cylinder $(h_2) = 60\ cm$

Radius of larger cylinder (r_1) = $\frac{24}{2}\ cm$

Radius of smaller cylinder (r_2) = 8 cm

We know that

Volume of larger cylinder = $\pi r^2{}_1 h_1$.

= $\pi \times \frac{24}{2} \times \frac{24}{2} \times 220$

= $\pi \times 144 \times 220\ cm^3$

and volume of smaller cylinder

= $\pi r_2{}^2{}_{h_2}$

= $\pi \times (8)^2 \times 60$

= $\pi \times 64 \times 60 cm^2$

Therefore, total volume of pole = Volume of larger cylinder +Volume of smaller cylinder

= $\pi \times 144 \times 220 + \pi \times 60 \times 64$

= $\pi(144 \times 220 + 64 \times 60)$

= $\pi(31680 + 3840)$ = 35520 $\pi\ cm^3$

Mass of the pole (at the rate of 8 gm per 1 cm^3) = Volume ×Density

= $35520 \times 3.14 \times 8 g$

= $\frac{35520\times314\times8}{1000\times100} kg$

= $\frac{8922624}{10000}$ = 892.260 kg

25. The following table gives the literacy rate (in %) of 35 cities. Find the mean literacy rate. [5]

Literacy rate (%)	**45-55**	**55-65**	**65-75**	**75-85**	**85-95**
No. of cities	**3**	**10**	**11**	**8**	**3**

Ans.

Literacy rate (in %)	**Number of cities (f_i)**	**Class mark (x_i)**	$u_i = \frac{x_i - 70}{10}$	$f_i u_i$
45-55	3	50	-2	-6
55-65	10	60	-1	-10
65-75	11	70 = a	=0	0
75-85	8	80	=1	8
85-95	3	90	=2	6

	$\sum f_i = 35$			$\sum f_i u_i = -2$

Here, we have $a = 70, h = 10 \sum f_i = 35$ $and \sum f_i u_i = -2$

Using step-deviation method,

Mean = $a + \frac{\sum f_i u_i}{\sum f_i} \times h$

$= 70 + \frac{(-2)}{35} \times 10$

$= 70 - \frac{4}{7}$

$= 70 - 0.57$

$= 69.43\%$

Hence, the mean literacy rate = 69.43%.