SECTION-A

1. Find HCF of 196 and 38220 using Euclid's divisionAlgorithm. $\left[2\frac{1}{2}\right]$

Ans. 38220 > 196, we apply the division lemma to 38220 and 196, to get

38220 = 196 × 195 + 0

The remainder now becomes zero, so our procedure stops.

Since the divisor at this stage is 196. So the HCF of 38220 and 196 is 196.

2. If HCF (306, 657) = 9 then find the LCM (306, 657). $\left[2\frac{1}{2}\right]$

Ans. We know that

$LCM\ (a, b) \times HCF\ (a, b) = a \times b$

$\Rightarrow LCM\ (a, b) = \frac{a \times b}{HCF\,(a,b)}$

$\Rightarrow LCM\ (306, 657) = \frac{306 \times 657}{9}$

$\Rightarrow LCM(306, 657) = 22338.$

3. Solve the following pair of Linear equation $\left[2\frac{1}{2}\right]$

$s - t = 3$

${}^{s}/_{3} + {}^{t}/_{2} = 6$

Ans. Given pair of linear equations

$s - t = 3$ ....(i)

$\frac{s}{3} - \frac{t}{2} = 6$ ....(ii)

From equation (i) , we get

$s = t + 3$ ....(iii)

Substituting this value s in equation (ii), we get

$\frac{t+3}{3} + \frac{t}{2} = 6$

$\Rightarrow 2(t + 3) + 3t = 36$

$\Rightarrow 5t + 6 = 36 \Rightarrow \quad t = 6.$

Substituting the value t in equation (iii), we get

$$s + 6 + 3 = 9$$

Hence, $s = 9, t = 6.$

4. Find the roots of quadratic $6x^2 - x - 2 = 0$. $\left[2\frac{1}{2}\right]$

Ans. $6x^2 - x - 2 = 0$

By spitting the middle term $-x$ as $-4x + 3x$.

$6x^2 - 4x + 3x - 2 = 0$

$2x(3x - 2) + 1(3x - 2) = 0$

$(3x - 2)(2x + 1) = 0$

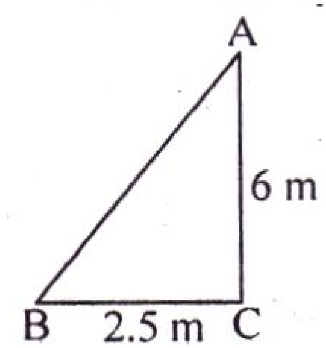
$3x - 2 = 0$ $\quad$ $2x + 1 = 0$

$3x = 2$ $\quad$ $2x = -1$

$x = \frac{2}{3}$ $\quad$ $x = \frac{-1}{2}$

5. A ladder is placed against a wall such that its foot is at a distance of 2.5 m from the wall and its top reaches a window 6 mabove the ground. Find the height (length) of the ladder. $\left[2\frac{1}{2}\right]$

Ans. Let's assume that AB is a ladder and CA is a wall. The window is on a point A.



In figure, BC = 2.5 m, CA = 6m

In right ΔABC,

$AB^2 = BC^2 + CA^2$

$AB^2 = (2.5)^2 + (6)^2$

AB^2 = 6.25 + 36 = 42.25.

$\Rightarrow AB$ = 6.5 m

Hence, length of the ladder = 6.5m

6. Find the point on x –axis which is equidistant from $(2, -5)$ *and* $(-2, 9)$. $\left[2\frac{1}{2}\right]$

Ans. Let the coordinates of $A(2, -5)$ and $B(-2, 9)$ are given points.

and $P(x, 0)$ be the required point on the x –axis such that

PA = PB

$PA = \sqrt{(2 - x)^2 + (-5 - 0)^2}$

$= \sqrt{4 + x^2 - 4x + 25}$

$= \sqrt{x^2 - 4x + 29}$

and $PB = \sqrt{(-2 - x)^2 + (9 - 0)^2}$

$= \sqrt{4 + x^2 + 4x + 81}$

$= \sqrt{x^2 + 4x + 85}$

It is given that PA = PB

$\Rightarrow \sqrt{x^2 - 4x + 29} = \sqrt{x^2 + 4x + 85}$

$\Rightarrow x^2 - 4x + 29 = x^2 + 4x + 85$

$\Rightarrow -4x + 29 = 4x + 85$

$\Rightarrow -4x - 4x = 85 - 29$

$\Rightarrow -8x = 56$

$\Rightarrow x = -7$

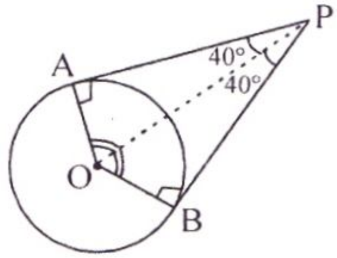
Hence, point on $x - axis = -7$

Therefore, the point equidistant from the given points is (-7, 0).

7. If tangents PA and PB from a point P to a circle with centre O are inclined to each other at angle 80^0, then find the value of $\angle POA$. $\left[2\frac{1}{2}\right]$

Ans. From the fig., we have

$\Delta AOP \cong \Delta OBP$ $(\because By\ SSS\ congruence)$



$\Rightarrow \angle POA = \angle POB = \frac{1}{2}\angle AOB$ ……………………….(i)

We know that

$\angle AOB + \angle APB = 180^0$

$\Rightarrow \angle AOB + 80^0 = 180^0$

$\Rightarrow \angle AOB = 180^0 - 80^0 = 100^0$ ………………………..(ii)

From (i) and (ii), we get.,

$\angle POA = \frac{1}{2} \times 100 = 50^0$

Hence, $\angle POA = 50^0$.

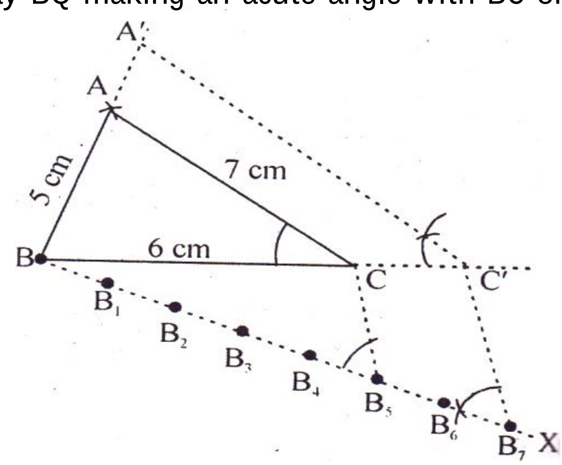
8. **Construct a triangle with sides 5 cm, 6 cm, 7 cm amd then another triangle whose sides are $\frac{7}{5}$ of the corresponding sides of first triangle.** $\left[2\frac{1}{2}\right]$

Ans. We are given the sides of triangle 5 cm, 6 cm and 7 cm and we are required to construct another triangle whose sides are $\frac{7}{5}$ of the corresponding sides of first triangle.

Steps of construction:

(i) Construct a triangle ABC in which AB = 5 cm, BC = 6 cm and CA =7cm.

(ii) Draw any ray BQ making an acute angle with BC on the side opposite to the vertex A.



(iii) Marks 7 points (the greater of 7 and 5 in 7/5) $B_1, B_2, B_3, B_4, B_5, B_6$ *and* B_7 So that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5 = B_5B_6 = B_6B_7$.

(iv) Join B_5C.

(v) Through B_7, draw $B_7C'||B_5C$ to intersect BC at C'.

(vi) Through C', draw C'A'|| CA to intersect BA at A' Now $\Delta A'BC'$ is the required triangle whose sides are 7/5 times the corresponding sides of the ΔABC.

Justification

$\Delta ABC \sim A'BC'$ (By AA similarity criteria)

Therefore, $\frac{BA'}{BA} = \frac{BC'}{BC} = \frac{A'C'}{AC}$

But $\frac{BC'}{BC} = \frac{7}{5} \left(\because \frac{BB_7}{BB_5} = \frac{BC'}{BC} = \frac{7}{5}\right)$

So, $\frac{BA'}{BA} = \frac{BC'}{BC} = \frac{A'C'}{AC} = \frac{7}{5}$

9. Gopi buys a fish from a shop. If there are 5 male and 8 female fishes in the tank what will be probability of a fish taken out randomly to be a male fish? $\left[2\frac{1}{2}\right]$

Ans. Number of male fish = 5

Number of female fish = 8

Then total number of fishes in the tank =5+8=13

$\therefore$ Probability of getting a male fish = $\frac{\text{possible outcomes}}{\text{Total outcomes}} = \frac{5}{13}$.

10. One card is drawn from a well shuffled deck of 52 cards. Find the probability of getting a king of red colour. $\left[2\frac{1}{2}\right]$

Ans. There are two red kings, (one each of diamond and heart)

So, number of favourable outsomes = 2

$\therefore$ Probability off getting king of red colour $= \frac{2}{52} = \frac{1}{26}$

SECTION-B

11. Divide $(3x^4 + 5x^3 - 7x^2 + 2x + 2)$ by $(x^2 + 3x + 1)$. $\left[3\frac{1}{2}\right]$

Ans. Here, dividend and divisor are in the standard form.

$$\begin{array}{r} 3x^2 - 4x + 2 \\ x^2 + 3x + 1 \overline{\smash{)}\, 3x^4 + 5x^3 - 7x^2 + 2x + 2} \\ 3x^4 + 9x^3 + 3x^2 \qquad\qquad \\ - \quad - \quad - \qquad\qquad \\ \hline -4x^3 - 10x^2 + 2x + 2 \\ -4x^3 - 12x^2 - 4x \quad \\ + \quad + \quad + \quad \\ \hline 2x^2 + 6x + 2 \\ 2x^2 + 6x + 2 \\ - \quad - \quad - \\ \hline 0 \end{array}$$

Since, remainder is zero, hence $x^2 + 3x + 1$ is a factor of $3x^4 + 5x^3 - 7x^2 + 2x + 2$.

12. Solve the pair of equations graphically: $\left[3\frac{1}{2}\right]$

$$x + 3y = 6$$

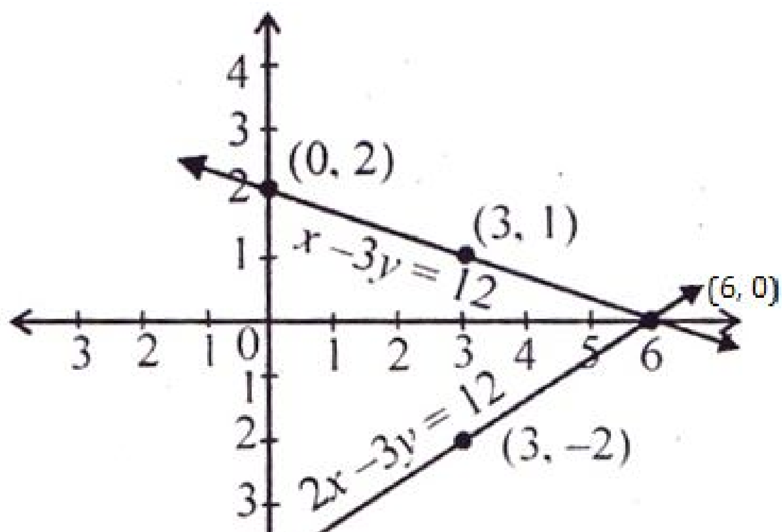
$$2x - 3y = 12$$

Ans. **Graphically representation:**

From equation (i), we get

x	6	3	0
y	0	1	2

$x + 6 - 3y$



From equation (ii), we get

$2x - 3y = 12$

$\Rightarrow \quad x = \frac{12+3y}{2}$

x	6	3	0
y	0	−2	−4

From the graph we find that lines are intersecting lines and have unique solution.

Hence, $x = 6, y = 0$.

13. How many multiples of 4 lie between 10 and 250? $\left[3\frac{1}{2}\right]$

Ans. The multiples of 4 between 10 and 250 are 12, 16, 20, 24 ……………………, 248.

So, the above sequence from an A.P.

Let these number = n.

$a_1 = 12,$

$d = a_2 - a_1$

$= 16 - 12 = 4$

$a_n = 248.$

We know that

$a_n = a + (n-1)d$

$\Rightarrow 248 = 12 + (n-1)4$

$\Rightarrow 248 = 12 + 4n - 4$

$\Rightarrow 4n = 248 - 8$

$\Rightarrow n = \frac{240}{4}$

$\Rightarrow n = 60$

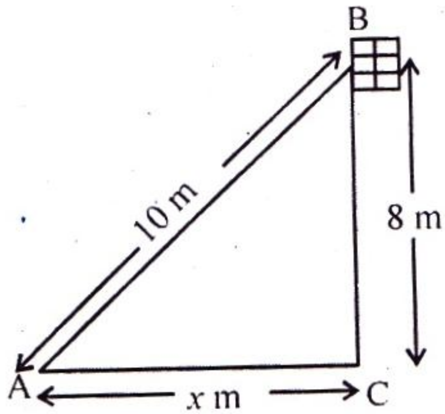
Hence, there are 60 terms lies multiple of 4 betwene 10 and 250.

14. A ladder 10 m long reaches a window 8m above the ground. Find the distance of the foot of ladder from base of wall. $\left[3\frac{1}{2}\right]$

Ans. Length of the ladder = 10 m

The height between a window and ground = 8 m

Let's assume the distance between foot of the ladder and base of the ball = xm



In right ΔABC,

$(AB)^2 = (BC)^2 + (AC)^2$ (Pythagoras Theorem)

$\Rightarrow (10)^2 = (8)^2 + (x)^2$

$\Rightarrow (x)^2 = (10)^2 - (8)^2$

$\Rightarrow x^2 = 100 - 64 = 36$

$\Rightarrow x = 6$

Hence, the distance between foot of the ladder and base of the wall = 6m

15. **Prove that :** $(\boldsymbol{cosec\,\theta - \cot\theta})^2 = \frac{\mathbf{1-\cos\theta}}{\mathbf{1+\cos\theta}}$. $\left[3\frac{1}{2}\right]$

Ans. $L.H.S = (cosec\,\theta - \cot\theta)^2$

$= \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2 = \left(\frac{1-\cos\theta}{\sin\theta}\right)^2$

$= \frac{(1-\cos\theta)^2}{sin^2\theta} = \frac{(1-\cos\theta)^2}{1-cos^2\theta}$ $[\because sin^2\theta = 1 - cos^2\theta]$

$= \frac{(1-\cos\theta)^2}{(1)^2-(\cos\theta)^2}$

$= \frac{(1-\cos\theta)(1-\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}$ $[\because a^2 - b^2 = [(a-b)(a+b)]$

$= \frac{1-\cos\theta}{1+\cos\theta} = R.H.S.$

$\therefore$ L.H.S = R.H.S.

16. **Prove that:** $\mathbf{\tan\ 48^0 \tan\ 23^0 \tan\ 42^0 \tan\ 67^0 = 1}$. $\left[3\frac{1}{2}\right]$

Ans. $L.H.S.$ = $\tan\ 48^0 \tan\ 23^0 \tan\ 42^0 \tan\ 67^0$

$\Rightarrow \tan\, 48^0 \tan\, 23^0 \tan(\,90^0 - 48^0) \tan(\,90^0 - 23^0)$

$\Rightarrow \tan\, 48^0 \tan\, 23^0 \cot\, 48^0 \cot\, 23^0$

$\Rightarrow \tan\, 48^0 \tan\, 23^0 \cot\, 48^0 \cot 23^0$

$\Rightarrow \tan\, 48^0 \tan\, 23^0 \frac{1}{\tan 48^0} \frac{1}{\tan 23^0} = 1$

Hence, L.H.S = R.H.S.

17. Find the area of a sector of a circle with radius 6 cm. If angle of the sector is 60^0. $\left[3\frac{1}{2}\right]$

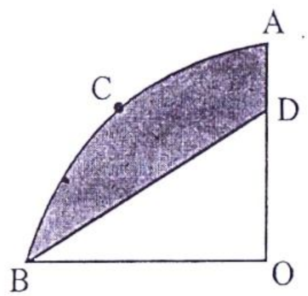
Ans. Radius of a circle (r) = 6 cm

Angle of the sector $(\theta) = 60^0$

Area of the sector $= \frac{\theta}{360} \times \pi r^2$

$= \frac{60}{360} \times \pi \times 6 \times 6 = \frac{132}{7} cm^2$.

18. OACB is a quadrant of a circle with centre O and radius 3.5 cm. If OD = 2 cm then find the area of quadrant and area of shaded region. $\left[3\frac{1}{2}\right]$



Ans. The radius of a quadrant OACB 3.5 cm and OD= 2 cm

The area of the quadrant OACB

$= \frac{1}{4} \times \pi r^2$

$= \frac{1}{4} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2 cm^2$

$= \frac{1}{4} \times \frac{22}{7} \times \frac{49}{4}\ cm^2$

$= \frac{11 \times 7}{8} cm^2$

$= \frac{77}{8} cm^2$

ΔOBD is a right angled triangle

Here, $OB = \frac{7}{2}\ cm\ and\ OD = 2\ cm$

The area of $\Delta OBD = \frac{1}{2}\ OB \times OD$

$= \frac{1}{2} \times \frac{7}{2} \times 2\ cm^2$

$= \frac{7}{2}\ cm^2$

Hence, the area of the shaded region = The area of quadrant OACB – The area of a $\Delta OBD = \frac{77}{8}\ cm^2 - \frac{7}{2}\ cm^2$

$= \frac{77-28}{8}\ cm^2 = \frac{49}{8}\ cm^2$.

19. **Find the area of a triangle whose vertices are $(-5,-1), (3,-5)$ and $(5,2)$.** $\left[3\frac{1}{2}\right]$

Ans. Let vertices of a given triangle ABC are $A(-5,-1), B(3,-5) and\ C\ (5,2)$.

Here, $x_1 = 5\ and\ y_1 = -1$

$x_2 = 3\ and\ y_2 = -5$

$x_3 = 5\ and\ y_3 = 2$

We Know that area of $\Delta ABC = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}[-5(-5-2) + 3(2-(-1)) + 5(-1-(-5))]$

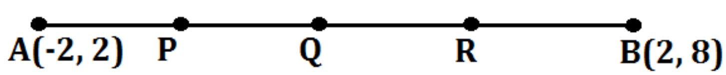
$= \frac{1}{2}[-5 \times -7 + 3 \times 3 + 5 \times 4]$

$= \frac{1}{2}[35 + 9 + 20]$

$= \frac{1}{2} \times 64 = 32$ square unit.

20. **Find the co-ordinates of the points which divides the line segment joining $A(-2,2)$ and B $(2,8)$ into four equal parts.** $\left[3\frac{1}{2}\right]$

Ans. Let the points P, Q and R divides the line segment $A(-2,2)$ and $B\ (2,8)$ into four equal parts.



$\therefore\ AP = PQ = QR = RB$

Now Q is mid point of AB.

By mid-point formula the co-ordinates of the point Q are $\left(\frac{-2+2}{2}, \frac{2+8}{2}\right)$ = (0.5)

R is mid-point of QB.

Then the coordinates of R are $\left(\frac{0+2}{2}, \frac{5+8}{2}\right) = \left(1, \frac{13}{2}\right)$

P is mid-point of AQ.

Then, the coordinates of P are

$\left(\frac{-2+0}{2}, \frac{2+5}{2}\right) = \left(-1, \frac{7}{2}\right)$

Hence, the required points are $P\left(-1, \frac{7}{2}\right)$, $Q(0, 5)$ and R $\left(1, \frac{13}{2}\right)$.

SECTION-C

21. An express train takes one hour less than a passenger train to travel 132 m between Mysore and Bangalore. If speed of express train is 11 km/h more than the passenger train then find the average speed of both trains. [5]

Ans. Let the average speed of the passenger train = $x\ km/h$

And the average speed of the express train = $y\ km/h$

Distance between mysore and Banagalore = 132 km.

Time taken by passenger train to travel a distance 132 km = $\frac{132}{x} hrs.$

Time taken by express train to travel a distance 132 km = $\frac{132}{x+11} hrs.$

According to question, $\frac{132}{x} - \frac{132}{x+11} = 1$

$\Rightarrow 132\left[\frac{1}{x} - \frac{1}{x+11}\right] = 1$

$\Rightarrow 132\left[\frac{x+11-x}{x(x+11)}\right] = 1$

$\Rightarrow \quad \frac{11}{x^2+11x} = \frac{1}{132}$

$\Rightarrow x^2 + 11x = 11 \times 132$

$\Rightarrow x^2 + 11x - 1452 = 0$

which is a quadratic equation in x variable

Here, $a = 1, b = 11, c = -1452$

$D = b^2 - 4ac$

$= (11)^2 - 4 \times 1 \times (-1452)$

$= 121 + 5808 = 5929$

Now, $x = \frac{-b \pm \sqrt{D}}{2a}$

$= \frac{-11 \pm \sqrt{5929}}{2 \times 1}$

$= \frac{-11 \pm 77}{2}$

$= \frac{66}{2}, \frac{-88}{2}$

$= 33, -44$

But speed of the train can't be negative.

So, $x = 33$

Hence, average speed of the passengers train $= 33\ km/h$

And average speed of the express train $= (33 + 1)\ km$

$= 44\ km/h$

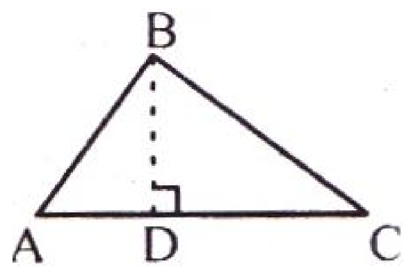
22. **In a right triangle, the square of the hypotenuse is kequal to the sum of the squares of the other two sides.** **[5]**

Ans. **Given**

A right angled ΔABC which is right angled at B.

To prove

$AC^2 = AB^2 + BC^2$



Construction

We draw, $BD \perp AC$

Proof

$\Delta ADB \sim \Delta ABC$

($\therefore$ Perpendicular is drawn from the vertex of the right angle of right triangle to the hypotenuse then triangle on both side of the perpendicular are similar to whole triangle)

$\Rightarrow \frac{AD}{AB} = \frac{AB}{AC}$ [$\because$ sides are proportional]

Or $AB \times AB = AD \times AC$

Or $AB^2 = AD \times AC$ ....(i)

Also, $\Delta CDB \sim \Delta CBA$

Therefore, $\frac{CD}{BC} = \frac{BC}{CA}$ [∵ sides are proportional]

Or $BC \times BC = CD \times CA$

Or $BC^2 = CD \times CA$ ....(ii)

Now, adding (i) and (ii), we get

$AB^2 = BC^2 = AD \times AC + CD \times AC$

$= AC[AD + CD]$

$= AC \times AC = AC^2.$

23. As observed from 75 m high light house from the sea level, the angle of depression of two ships are 30^0 *and* 45^0. If one ship is exactly behind the other on the same side of the lighthouse find the distance between the two ships. [5]

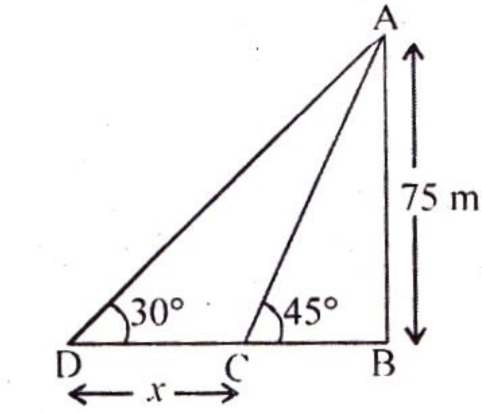
Ans. Let the distance between the ships = $x\ m$

Height of the light house (AB) = 75 m,

D and C are the positions of two ships.

$\angle ADB = 30^0$ (Given)

$\angle ACB = 45^0$ (Given)



In right ΔADB,

$\tan 30^0 = \frac{AB}{DB}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{75}{DB}$

$\Rightarrow \quad DB = (75 \times \sqrt{3})$ ………………(i)

In right ΔABC,

$\tan 45^0 = \frac{AB}{BC}$

$\Rightarrow \quad 1 = \frac{75}{BC}$

$\Rightarrow \quad BC = 75m$ …………………(ii)

Now the distance between the ships

$x = DB - BC = 75\sqrt{3} - 75 = 75(\sqrt{3} - 1)m.$

24. **How many silver coins, 1.75 cm in diameter and thickness of 2 mm must be melted to form a cuboid of dimensions of $5.5\ cm \times 10cm \times 3.5\ cm$?** **[5]**

Ans. Given

Diameter of the silver coin = 1.75 cm

Radius of the silver coin (r) = $\frac{1.75}{2} cm$

Thickness of the silver coin = 2 mm

= 0.2 cm

Dimensions of cuboid = 5.5 cm × 10cm × 3.5 cm

Let the numbers of coin = n

We know that

Volume of n coins = Volume of the cuboid

$n \times \pi r^2 h = l \times b \times h$

$\Rightarrow \quad n \frac{22}{7} \times \left(\frac{1.75}{2}\right)^2 \times (0.2) = 5.5 \times 10 \times 3.5$

$\Rightarrow \quad n \times \frac{22}{7} \times \frac{7}{7} \times \frac{7}{8} \times 2 = 55 \times 35$

$\Rightarrow \quad n = \frac{55 \times 35 \times 16}{7 \times 11}$

$\Rightarrow \quad n = 5 \times 5 \times 16 = 400.$

The number of coins = 400.

25. **The distribution below gives the weights of 30 students of a class. Find the median weight of the students.** **[5]**

Weight (in kg)	40-45	45-50	50-55	55-60	60-65	65-70	70-75
Number of students	2	3	8	6	6	3	2

Ans.

Weight (in kg)	**No. of student** (f_1)	**Cumulative frequency** $(c.f.)$
40-45	2	2
45-50	3	2+3=5
50-55	8	5+8=13
55-60	6	13+6=19
60-65	6	19+6=25
65-70	3	25+3=28
70-75	2	28+2=30

Here, we have, n=30

So, $\frac{n}{2} = 15$, therefore the median class is $55 - 60$.

Here $l = 55, n = 30, f = 6, c.f. = 13$ and $h = 5$

Using the formula of median,

Median = $l + \left[\frac{\frac{n}{2} - c.f.}{f}\right] \times h$

$= 55 + \left[\frac{15-13}{6}\right] \times 5 = 55 + \frac{5}{3} = 55 + 1.67 = 56.67\ kg$

Hence, median weight of the students = 56.67 kg.