Q.1 **The principal value of $sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ is :** **[1]**

(a) $\frac{\pi}{6}$ **(b)** $\frac{\pi}{2}$

(c) $\frac{\pi}{3}$ **(d)** $\frac{\pi}{4}$

Solution : (d)

Q.2 **If A and B are symmetric matrices of same order then AB - BA is :** **[1]**

(a) **skew symmetric matrix**

(b) **symmetric matrix**

(c) **zero matrix**

(d) **identity matrix**

Solution : (a)

Q.3 **The derivative of a^x is :** **[1]**

(a) a^x **(b)** $\frac{a^x}{log\ a}$

(c) $a^x log\ a$ **(d)** **None of these**

Solution : (c)

Q.4 **The function f (x) = log x is strictly increasing on :** **[1]**

(a) $[0, \infty]$ **(b)** $(0, \infty)$

(c) $(-\infty, \infty)$ **(d)** **None of these**

Solution : (b)

Q.5 $\int e^x\ sec\ x\ (1 + tanx)$ **dx is equal to :** **[1]**

(a) $e^x cos\ x + c$ **(b)** $e^x sec\ x + c$

(c) $e^x sin\ x + c$ **(d)** $e^x tan\ x + c$

Solution : (b)

Q.6 **The degree of differential equation**

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + sin\ y = 0$$ **[1]**

(a) **1** **(b)** **2**

(c) **3** **(d)** **Not defined**

Solution : (a)

Q.7 **The scalar product is commutative if :** **[1]**

(a) $\vec{a}.\vec{b} = -\vec{b}.\vec{a}$ **(b)** $\vec{a} \times \vec{b} = \vec{b} \times \vec{a}$

(c) $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$ **(d)** $\vec{a}.\vec{b} = \vec{b}.\vec{a}$

Solution : (d)

Q.8 **Angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes 1 and 2 respectively and when $\vec{a}.\vec{b} = 1$** **[1]**

(a) 3 **(b)** $\frac{\pi}{2}$

(c) $\frac{\pi}{3}$ **(d)** $\frac{\pi}{4}$

Solution : (c)

Q.9 **If a line makes an angle of $\frac{\pi}{4}$ with each of y- and z- axis, then the angle which it makes with x-axis is :** **[1]**

(a) $\frac{3\pi}{2}$ **(b)** $\frac{\pi}{2}$

(c) $\frac{\pi}{4}$ **(d)** $\frac{3\pi}{2}$

Solution : (b)

Q.10 **If A and B are two independent events and P(A) = $\frac{3}{5}$, P(B) = $\frac{1}{5}$, then P(A$\cap B$) is :** **[1]**

(a) $\frac{3}{5}$ **(b)** $\frac{3}{25}$

(c) $\frac{1}{25}$ **(d)** $\frac{1}{5}$

Solution : (b)

Q.11 **Using elementary operations, find the inverse of matrix A = $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$** **[2]**

Solution: In order to use elementary row operations, we may write $A = IA$

or $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

$\begin{bmatrix} 1 & -1 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A$ (applying $R_2 \to R_2 - 2R_1$)

or $\begin{bmatrix} \mathbf{1} & \mathbf{-1} \\ \mathbf{0} & \mathbf{1} \end{bmatrix} = \begin{bmatrix} \mathbf{1} & \mathbf{0} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix} \boldsymbol{A}$ (applying $\boldsymbol{R_2} \to \frac{1}{5}\boldsymbol{R_2}$)

or $\begin{bmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{bmatrix} = \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix} \boldsymbol{A}$ (appling $\boldsymbol{R_1} \to \boldsymbol{R_1} + \mathbf{2}\boldsymbol{R_2}$)

$\Rightarrow \boldsymbol{A}^{-1} = \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix}$

or

For matrix

A = $\begin{bmatrix} \mathbf{1} & \mathbf{5} \\ \mathbf{6} & \mathbf{7} \end{bmatrix}$

Verify that $A + A'$ is symmetric matrix.

Solution : We have, A = $\begin{bmatrix} \mathbf{1} & \mathbf{5} \\ \mathbf{6} & \mathbf{7} \end{bmatrix} \Rightarrow \boldsymbol{A}' = \begin{bmatrix} \mathbf{1} & \mathbf{6} \\ \mathbf{5} & \mathbf{7} \end{bmatrix}$

Here, $\boldsymbol{A} + \boldsymbol{A}' = \begin{bmatrix} \mathbf{1} & \mathbf{5} \\ \mathbf{6} & \mathbf{7} \end{bmatrix} + \begin{bmatrix} \mathbf{1} & \mathbf{6} \\ \mathbf{5} & \mathbf{7} \end{bmatrix} = \begin{bmatrix} \mathbf{2} & \mathbf{11} \\ \mathbf{11} & \mathbf{14} \end{bmatrix}$

$(\boldsymbol{A} + \boldsymbol{A}')' = \begin{bmatrix} \mathbf{2} & \mathbf{11} \\ \mathbf{11} & \mathbf{14} \end{bmatrix} = (\boldsymbol{A} + \boldsymbol{A}')$

$\therefore$ $\boldsymbol{A} + \boldsymbol{A}'$ is symmetric.

which is a symmetric matrix.

Q.12 **Examine the function given by**

$\boldsymbol{f(x)} = \begin{cases} \boldsymbol{x^2 sin \frac{1}{x}, x \neq 0} \\ \mathbf{0}, \boldsymbol{x = 0} \end{cases}$ **for continuity.** **[2]**

Solution : $\boldsymbol{f(x)} = \begin{cases} \boldsymbol{x^2 sin\frac{1}{x}, if\ x \neq 0} \\ \mathbf{0}, \boldsymbol{if\ x = 0} \end{cases}$

$\lim_{x\to 0} \boldsymbol{f\,x} = \lim_{x\to 0} \left(\boldsymbol{x^2 sin\frac{1}{x}}\right)$

$= \mathbf{0} \cdot \boldsymbol{sin}\left(\frac{1}{0}\right)$

$= \mathbf{0}$ $[\because\ |\boldsymbol{sinx}| \leq \mathbf{1}]$

$\boldsymbol{Also\ f(0) = 0}$

$\therefore$ $\lim_{x\to 0} \boldsymbol{f(x)} = \boldsymbol{f(0)} = \mathbf{0}$

Therefore, f(x) is continuous for all x.

Q.13 **A balloon which always remains spherical on inflation, is being inflated by pumping in 900 cubic centimeter of gar per second. Find the rate at which the radius of the balloon increases when the radius is 15 cm.** **[2]**

Solution : Let 'r' be the radius and 'V' be the volume of the spherical balloon, then

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = \frac{d}{dr}\left(\frac{4}{3}\pi r^3\right]\frac{dr}{dt}$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$ ....(1)

According to Question,

$\frac{dV}{dt} = 900 cm^3/sec$

$\therefore 900 = 4\pi r^2 \frac{dr}{dt}$ ....(2) [using (1)]

when $r = 15$ then (2) becomes

$900 = 4\pi \times (15)^2 \frac{dr}{dt}$

$\Rightarrow \frac{dr}{dt} = \frac{900}{4\times\pi\times 15\times 15} = \frac{1}{\pi}$

Hence, the radius of the balloon is increasing at the rate of $\frac{1}{\pi}$ cm/sec.

Q.14 **From the differential equation of the family of parabolas having vertex at origan and axis along positive y-axis.** **[2]**

Solution: We know that the equation of family of parabolas having axis along positive y −axis and vertex at origin is given by

$x^2 = 4ay$ ....(1)

Where a is any arbitarary constant.

On differentiating equation (1) w.r.t. $'x'$:

$2x = 4ay_1 \Rightarrow x = 2ay_1$

$\Rightarrow a = \frac{x}{2y_1}$ ....(2)

Using the value of 'a' from equation (2) in equation (1), we have

$x^2 = 4\cdot\frac{x}{2y_1}\cdot y = \frac{2xy}{y_1} \Rightarrow x = \frac{2y}{y_1}$

$\Rightarrow xy_1 = 2y \quad \Rightarrow xy_1 - 2y = 0$

Which is required differential equation.

Q.15 **Find gof and fog, if $f(x) = |x|$ and $g(x) = |5x - 2|$.** $\left[3\frac{1}{2}\right]$

Solution : Here, $f(x) = |x|$ and $g(x) = |5x - 2|$

Then $gof = g(f(x)) = g(|x|) = |5|\,x\,|-2|$

and $fog = f(g(x)) = f(|5x - 2|) = ||5x - 2|| = |5x - 2|$.

Q.16 **Solve $tan^{-1}\,2x + tan^{-1}\,3x = \frac{\pi}{4}$** $\left[3\frac{1}{2}\right]$

Solution: We have $tan^{-1}2x + tan^{-1}3x = \frac{\pi}{4}$

or $tan^{-1}\left(\frac{2x+3x}{1-2x\times 3x}\right) = \frac{\pi}{4}$

i.e. $tan^{-1}\left(\frac{5x}{1-6x^2}\right) = \frac{\pi}{4}$

Therefore $\frac{5x}{1-6x^2} = \tan\frac{\pi}{4} = 1$

$6x^2 + 5x - 1 = 0$ i.e., $(6x - 1)(x + 1) = 0$

Which gives $x = \frac{1}{6}$ or $x = -1$

Since $x = -1$ does not satisfy the equation, as the L.H.S. of the equation becomes negative, $x = \frac{1}{6}$ is the only solution of the given equation.

OR

Express for following in the simplest form :

$tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right), x < \pi$

Solution: We write, $tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right) = tan^{-1}\left(\sqrt{\frac{2sin^2\frac{x}{2}}{2cos^2\frac{x}{2}}}\right)$

$= tan^{-1}\left(\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}\right) = \frac{x}{2}$.

Q.17 **Prove that $\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix} = (a-b)(b-c)(c-a)$** $\left[3\frac{1}{2}\right]$

Solution : Applying $R_2 \rightarrow R_2 - R_1$ and $R_3 \rightarrow R_3 - R_{1,}$ we get

$$\Delta = \begin{vmatrix} 1 & a & bc \\ 0 & b-a & c(a-b) \\ 0 & c-a & b(a-c) \end{vmatrix}$$

Taking factors (b-a) and (c-a) common from R_2 $and R_{3,}$ respectively, we get

$$\Delta = (b-a)(c-a)\begin{vmatrix} 1 & a & bc \\ 0 & 1 & -c \\ 0 & 1 & -b \end{vmatrix}$$

$= (b-a)(c-a)[(-b+c)]$ $(Expanding\ along\ first\ column)$

$= (a-b)(b-c)(c-a).$

Q.18 **Differenbtiate $(\boldsymbol{log\ x})^{\boldsymbol{cos\ x}}$** $\left[3\frac{1}{2}\right]$

Solution : Let y = $(logx)^{\cos x}$

Taking logarithm on both sides, we have

$(\log y = \cos x \cdot \log(\log x)$

Now, differentiating both sides ,we get

$\frac{1}{y} \cdot \frac{dy}{dx} = \cos x \cdot \frac{1}{(\log x)} \cdot \frac{d}{dx}(\log x) + log(\log x\ )(-\sin x)$

$= \frac{\cos x}{\log x}\left(\frac{1}{x}\right) - \sin x \cdot log(\log x)$

$\Rightarrow \frac{dy}{dx} = y\left[\frac{\cos x}{x\log x} - sinx \cdot log(logx)\right]$

$or\ \ \frac{dy}{dx} = (\log x)^{\cos x}\left[\frac{\cos x}{x\log x} - \sin x \cdot \log\,(\log x)\right].$

or

Find $\frac{dy}{dx}$ $\boldsymbol{if\ y^x = x^y}$

Solution : If $y^x = x^y$, $taking$ log $on\ both\ side$

$x\log y = y\log x$

$x \times \frac{1}{y}\frac{dy}{dx} + \log y = y \cdot \frac{1}{x} + \log x \cdot \frac{dy}{dx}$

$\frac{dy}{dx}\left(\frac{x}{y} - \log x\right) = \frac{y}{x} - \log y$

$\frac{dy}{dx} = \frac{\frac{y}{x} - \log y}{\frac{x}{y} - \log x}$

$\Rightarrow \qquad \frac{dy}{dx} = \frac{y}{x}\left(\frac{y-x\log y}{x-y\log x}\right).$

Q.19 **Evaluate** $\int \frac{x}{(x-1)^2(x+2)}dx$ $\left[3\frac{1}{2}\right]$

Solution : $\int \frac{x}{(x-1)^2(x+2)}dx$

Let $\frac{x}{(x-1)^2(x+2)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{c}{x+2}$

$\Rightarrow x = A(x-1)(x+2) + B(x+2) + C(x-1)^2$ ......(1)

$Put\ x = 0\ in\ eq.$ (1)

$0 = -2A + 2B + C$ ....(2)

$Now\ Put\ x-1 = 0 \Rightarrow x = 1\ in\ eq.$ (1)

$1 = 3B \Rightarrow B = \frac{1}{3}$

$Put\ x+2 = 0 \Rightarrow x = -2\ in\ eq.$ (1)

$-2 = C(-2-1)^2 = 9C \Rightarrow C = -\frac{2}{9}$

$\therefore From\ equation$ (2) :

$0 = -2A + \frac{2}{3} - \frac{2}{9}$

$-2A + \frac{4}{9} \Rightarrow 0$

$A = \frac{2}{9}$

$\therefore \frac{x}{(x-1)^2(x+2)} = \frac{-1}{18(x-1)} + \frac{1}{3(x-1)^2} - \frac{2}{9(x+2)}$

$I = \frac{2}{9}\int \frac{1}{x-1}dx + \frac{1}{3}\int (x-1)^{-2}dx - \frac{2}{9}\int \frac{1}{x+2}dx$

$= \frac{2}{9}log|x-1| + \frac{1}{3}\frac{(x-1)^{-1}}{-1} - \frac{2}{9}log|x+2| + c$

$I = \frac{2}{9}log|x-1| - \frac{1}{3(x-1)} - \frac{2}{9}log|x+2| + c$.

Q.20 **Evaluate** $\int \frac{5x+3}{\sqrt{x^2+4x+10}}dx$ $\left[3\frac{1}{2}\right]$

Solution : $I = \int \frac{5x+3}{\sqrt{x^2+4x+10}}dx$

$Let\ 5x+3 = A(2x+4) + B$

$5x + 3 = 2Ax + 4A + B$

On comparing coefficients, we have

$2A = 5,$ $\qquad 4A + B = 3$

$A = \frac{5}{2},$ $\qquad 4 \cdot \frac{5}{2} + B = 3$

$10 + B = 3$

$\Rightarrow \quad B = 3 - 10 = -7$

$\therefore 5x + 3 = \frac{5}{2}(2x + 4) - 7$

Now, I = $\int \frac{\frac{5}{2}(2x+4)-7}{\sqrt{x^2+4x+10}} dx$

$= \frac{5}{2}\int \frac{2x+4}{\sqrt{x^2+4x+10}} dx - \frac{7}{1}\int \frac{1}{\sqrt{x^2+4x+10}} dx$

$= \frac{5}{2}\int (x^2 + 4x + 10)^{-\frac{1}{2}} (2x + 4)dx - 7\int \frac{1}{\sqrt{(x+2)^2+10-4}} dx$

$= \frac{5}{2} \frac{(x^2+4x+10)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} - 7\int \frac{1}{\sqrt{(x+2)^2+(\sqrt{6})^2}} dx$

$= \frac{5}{2} \frac{\sqrt{x^2+4x+10}}{\frac{1}{2}} - 7log\left|x + 2 + \sqrt{x^2 + 4x + 10}\right| + c$

I $=5\sqrt{x^2 + 4x + 10}$ -7log $\left|x + 2 + \sqrt{x^2 + 4x + 10}\right| + c.$

Q.21 **Evaluate** $\int_0^{\pi} \frac{x \tan x}{\sec x+\tan x} dx$ $\left[3\frac{1}{2}\right]$

Solution : Let I $=\int_0^{\pi} \frac{\text{x tan x}}{\text{sec x+tan x}}$dx

$= \int_0^{\pi} \frac{(\pi-x)\tan(\pi-x)dx}{\sec(\pi-x)+\tan(\pi-x)}$ $\left[\text{using,} \int_0^{\alpha} f(x)dx = \int_0^{\alpha} f(a-x)dx\right]$

$\Rightarrow I = \int_0^{\pi} \frac{-(\pi-x)\tan x}{-\sec x-\tan x} dx$

$\Rightarrow I = \int_0^{\pi} \frac{\pi\tan x}{\sec x+\tan x} dx - \int_0^{\pi} \frac{x\tan x}{\sec x+\tan x} dx$

$\Rightarrow I = \int_0^{\pi} \frac{\pi\tan x}{\sec x+\tan x} dx - \text{I} \Rightarrow 2\text{I} = \int_0^{\pi} \frac{\pi\sin x\, dx}{1+\sin x} dx$

$= \int_0^{\pi} \frac{\pi\sin x}{1+\sin x} \times \frac{1-\sin x}{1-\sin x} dx = \int_0^{\pi} \frac{\pi(sinx-sin^2x)dx}{cos^2\text{x}}$

$= \int_0^{\pi} \frac{\pi\sin x}{cos^2\text{x}} dx - \int_0^{\pi} \frac{\pi\, sin^2x}{cos^2 x} dx$

$=\pi \int_0^{\pi} \tan x \sec x \, dx - \pi \int_0^{\pi} tan^2 x \, dx$

$= \pi[\sec x]_0^{\pi} - \pi \int_0^{\pi} (sec^2 x - 1) \, dx$

$= \pi[\sec \pi - \sec 0] - \pi[\tan x - x]_0^{\pi}$

$\Rightarrow 2I = \pi[-1 - 1] - \pi[tan\pi - tan0 - (\pi - 0] = -2\pi - \pi[-\pi]$

$\Rightarrow I = -\pi + \frac{\pi^2}{2} \Rightarrow I = \pi\left(\frac{\pi}{2} - 1\right).$

Q.22 **Solve the differential equation :**

$\left(x + 3y^2\right)\frac{dy}{dx} = y, (y > 0)$ $\left[3\frac{1}{2}\right]$

Solution : Given differential equation is $(x + 3y^2)\frac{dy}{dx} = y$

$\Rightarrow \frac{dy}{dx} = \frac{y}{x+3y^2}$

Taking its reciprocal, we get

$\frac{dx}{dy} = \frac{x+3y^2}{y} \Rightarrow \frac{dx}{dy} = \frac{x}{y} + 3y \Rightarrow \frac{dx}{dy} - \frac{1}{y}x = 3y$ ……(i)

Clearly, equation (i) is of the form $\frac{dx}{dy} + Rx = S$

Here, $R = -\frac{1}{y} and \; S = 3y$

Now, therefore I.F. $= e^{\int R dy} = e^{-\int \frac{1}{y} dy} = e^{-\log y}$.

$= e^{log\left(\frac{1}{y}\right)} = \frac{1}{y}$

Therefore, solution of equation(i) is

$x(I.F.) = \int (I.F.).S \, dy \Rightarrow x\left(\frac{1}{y}\right) = \int 3y\left(\frac{1}{y}\right) dy$

$\Rightarrow \frac{x}{y} = \int 3 \, dy \Rightarrow \frac{x}{y} = 3y + C \Rightarrow x = 3y^2 + cy;$

which is required general solution.

or

Solve the differential equation :

$(x^2 - y^2)dx = 2xy \, dy = 0.$

Solution: $(x^2 - y^2)dx = 2xy \, dy = 0.$

$$\frac{dy}{dx} = \frac{y^2-x^2}{2xy}$$

Putting, y = vx

$\Rightarrow \quad \frac{dy}{dx} = v + \frac{xdv}{dx}$

$\therefore \quad v + \frac{xdv}{dx} = \frac{v^2x^2-x^2}{2xvx} = \frac{v^2-1}{2v}$

$\Rightarrow \quad x\frac{dv}{dx} = \frac{v^2-1}{2v} - v = \frac{v^2-1-2v^2}{2v} = \frac{-1-v^2}{2v}$

$\Rightarrow \quad \frac{2v}{1+v^2}dv = \frac{-dx}{x}$

$\Rightarrow \quad \log|1 + v^2| = \log|x| + \log c$

$\Rightarrow \quad log\left|1 + \frac{y^2}{x^2}\right| = \log\left|\frac{c}{x}\right|$

$\Rightarrow \quad \log\left|\frac{x^2+y^2}{x^2}\right| = log\left|\frac{c}{x}\right| \Rightarrow \frac{x^2+y^2}{x^2} = \frac{c}{x} \Rightarrow x^2 + y^2 = cx.$

Q.23 **Find λ if the vectors $\vec{a} = \hat{\imath} + 3\hat{\jmath} + \hat{k}$, $\vec{b} = 2\hat{\imath} - \hat{\jmath} - \hat{k}$ and $\vec{c} = \lambda\hat{\imath} + 7\hat{\jmath} + 3\hat{k}$ are coplanar.** $\left[3\frac{1}{2}\right]$

Solution Since $\vec{a}, \vec{b}$ and $\vec{c}$ are coplanar vectors, we have $[\vec{a}, \vec{b}, \vec{c}] = 0$, i.e.,

$$\begin{vmatrix} 1 & 3 & 1 \\ 2 & -1 & -1 \\ \lambda & 7 & 3 \end{vmatrix} = 0$$

$\Rightarrow 1(-3 + 7) - 3(6 + \lambda) + 1(14 + \lambda) = 0$

$\Rightarrow \lambda = 0$

Q.24 **Find the vector and cartesian equation of planes that passes through the point $(1, 0, -2)$ and the normal to the plane is $\hat{\imath} + \hat{\jmath} - \hat{k}$.** $\left[3\frac{1}{2}\right]$

Solution: The equation of plane qhich passes through the point $(1, 0, -2)$ and normal to plane is

$(\hat{\imath} + \hat{\jmath} - \hat{k})$ given by

$[\vec{r} - (\hat{\imath} + 0\hat{\jmath} - 2\hat{k})] \cdot (\hat{\imath} + \hat{\jmath} - \hat{k}) = 0$

In cartesian form is

$[(x\hat{\imath} + y\hat{\jmath} + z\hat{k}) - (\hat{\imath} + 0\hat{\jmath} - 2\hat{k}) \cdot (\hat{\imath} + \hat{\jmath} - \hat{k}) = 0$

$\Rightarrow [(x-1)\hat{\imath} + y\hat{\jmath} + (z+2)\hat{k}] \cdot (\hat{\imath} + \hat{\jmath} - \hat{k}) = 0$

$\Rightarrow x - 1 + y - z - 2 = 0$

$\Rightarrow x + y - z - 3 = 0$

Q.25 **Two cards are drawn at random and without replacement from a pack of 52 playing cards. Find the probability that both the cards are back.** $\left[3\frac{1}{2}\right]$

Solution : Let $E_1 : first\ card\ drawn\ is\ back\ and$

$E_2 : second\ card\ drawn\ is\ also\ black$

Therefore, required probability = P$(E_{1)}\ P(E_2/E_{1)}$ $= \frac{26}{52} \times \frac{25}{51} = \frac{25}{102}$

(because, in 52 cards, there are 26 cards, which are black and when a card is drawn, then total no. of black cards left are 25.)

Q.26 **A die is thrown 6 times. If' getting an odd number is a success, what is probability of** $\left[3\frac{1}{2}\right]$

(i) 5 successes

(ii) at least 5 successes

(iii) at most 5 successes

Solution : Here, n=6 and success is getting an odd number

$\therefore p = \frac{3}{6} = \frac{1}{2}, and\ q = 1 - \frac{1}{2} = \frac{1}{2}$

(i) P(5 successes)= ${}^6C_5\ p^5q^1 = {}^6C_5 \left(\frac{1}{2}\right)^5 \left(\frac{1}{2}\right) = \frac{3}{32}$

(ii) P (atleast 5 successes) =P(5 successes) + P (6 successes)

$= {}^6C_5\ p^5q^1 = {}^6C_6\ p^6q^0$

$= \frac{3}{32} + 1\left(\frac{1}{2}\right)^6 = \frac{3}{32} + \frac{1}{64} = \frac{7}{64}$

(iii) P (atmost 5 successes) = 1-P(6 successes)

$=1 - \frac{1}{64} = \frac{63}{64}$.

or

If A and B are two events such that P(A)=$\frac{1}{2}$, P(B)=$\frac{7}{12}$ and P (not A or not B)=$\frac{1}{4}$, state whether A and B are independent?

Solution : Given, $P(not\ A\ or\ not\ B) = \frac{1}{4}$

$\Rightarrow P(A^C\ or\ B^C) = \frac{1}{4}$ $\Rightarrow P(A^C \cup B^C) = \frac{1}{4}$

$\Rightarrow P((A \cap B)^C) = \frac{1}{4}$

$\Rightarrow 1 - P(A \cap B) = \frac{1}{4}$

$\Rightarrow P(A \cap B) = 1 - \frac{1}{4} = \frac{3}{4}$

Also, $P(A) \cdot P(B) = \frac{1}{2} \times \frac{7}{12} = \frac{7}{12} \neq \frac{3}{4}$

$\Rightarrow P(A) \cdot P(B) \neq P(A \cap B)$.

Therefore, A and B are not independent.

Q.27 **Solve the following equations Matrix method :** **[5]**

$$\boldsymbol{x - y + z = 4}$$

$$\boldsymbol{2x + y - 3z = 0}$$

$$\boldsymbol{x + y + z = 2}$$

Solution : Given system of equations can be written as AX = B;

where, $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$ $and\ X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

Now, $|A| = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{vmatrix}$

$$= 1(1+3) + 1(2+3) + 1(2-1) = 4 + 5 + 1$$

$$|A| = 10$$

Since $|A| \neq 0, therefore, A^{-1} exists$

Now, adj A $= \begin{bmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix}' = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$

and $A^{-1} = \frac{adj\ A}{|A|} = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$

$Now, X = A^{-1}B = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 16+4 \\ -20+10 \\ 4+6 \end{bmatrix}$

$$= \frac{1}{10}\begin{bmatrix} 20 \\ -10 \\ 10 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

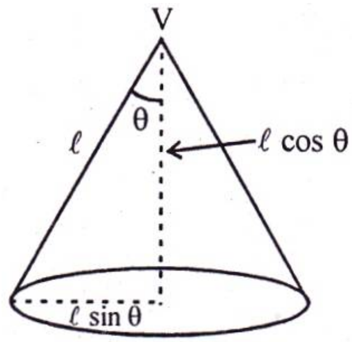
Therefore, required solution is $x = 2, y = -1, z = 1$.

Q.28 **Show that the semi-vertical angle of the cone of maximum volume and of given slant height $\boldsymbol{tan^{-1}\sqrt{2}}$** **[5]**

Solution : Let θ is the semi- vertical angle, ℓ is the given slant height . then radius of base $= \ell$ sinθ,

height $= \ell\ sin\theta, height = \ell \cos\theta$

and volume $v = \frac{1}{3}\pi(\ell \sin\theta)^2 \ell\cos\theta = \frac{1}{3}\pi\ell^3 \sin^3\theta\cos\theta$



$\Rightarrow \frac{dv}{d\theta} = \frac{1}{3}\pi\ell^3[\sin^2\theta(-\sin\theta) + \cos\theta(2\sin\theta\cos\theta)]$

$\Rightarrow \frac{dv}{d\theta} = \frac{1}{3}\pi\ell^3\sin\theta[-\sin^2\theta + 2(1-\sin^2\theta)]$

$\Rightarrow \frac{dv}{d\theta} = \frac{1}{3}\pi\ell^3\sin\theta\cos^2\theta[2(\sec^2\theta - \tan^2\theta) - \tan^2\theta]$

$\Rightarrow \frac{dv}{d\theta} = \frac{1}{3}\pi\ell^3\sin\theta\cos^2\theta\,[2 - \tan^2\theta] = 0$

For maximum volume, let $\frac{dv}{d\theta} = 0$

$\Rightarrow \frac{1}{3}\ell^3\sin\theta\cdot\cos^2\theta[2-\tan^2\theta] = 0$

$\Rightarrow \tan\theta = \sqrt{2} \Rightarrow \theta = \tan^{-1}\sqrt{2}$ (because,0< $\theta < \pi/2$)

Therefore, V is absolutely maximum for = $\tan^{-1}\sqrt{2}$.

or

Find points at which the tangent to the curve $y = x^3 - 3x^2 - 9x + 7$, is parallel to the x-axis.

Solution : $y = x^3 - 3x^2 - 9x + 7$

$\frac{dy}{dx} = 3x^2 - 6x - 9$

slope of tangent = $3x^2 - 6x - 9$

We know that when tangent is parallel to x –axis then slope is zero.

i.e. slope = 0

$\Rightarrow 3x^2 - 6x - 9 = 0$

$\Rightarrow x^2 - 2x - 3 = 0$

$\Rightarrow (x+1)(x-3) = 0$

$\Rightarrow x = -1, 3$

when $x = -1, y = (-1)^3 - 3(-1)^2 - 9(-1) + 7$

$= -1 - 3 + 9 + 7 = 12$

One point is $(-1, 12)$

Similarly when $x = 3$ then

$y = (3)^3 - 3(3)^2 - 9.3 + 7$

$= 27 - 27 - 27 + 7$

$y = -20$

$\therefore$ point $(3, -20)$

Q.29 **Find the area of region bounded by the curves : $4y = 3x^2$ and the line 2y = 3x +12.** **[5]**

Solution : Given parabola is $4y = 3x^2$ .....(i)

and the given line is $2y = 3x + 12$ ....(ii)

The two curves meet where

$2(3x + 12) = 3x^2$ (Eliminating y between (i) and (ii)

or $3x^2 - 6x - 24 = 0$

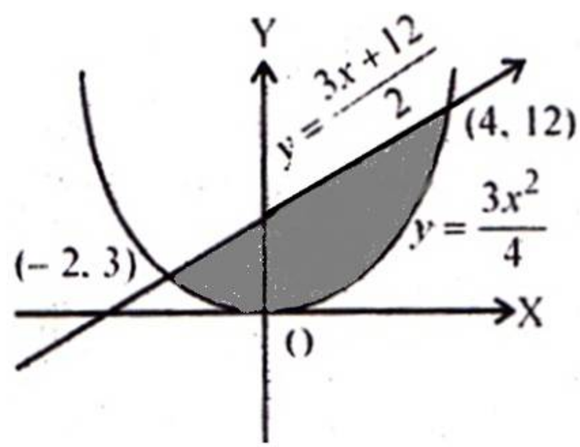
$or\ x^2 - 2x - 8 = 0$

$or\ (x-4)(x+2) = 0$

$\Rightarrow x = 4, -2$

When $x = 4,\ y = \frac{3\times4+12}{2} = 12\ and$

When $x = -2, y = \frac{3\times(-2)+12}{2} = 3$



Required area (shown shaded)

$= \int_{-2}^{4}\left(\frac{3x+12}{2} - \frac{3x^2}{4}\right)dx = \left[\frac{1}{2}\left(\frac{3x^2}{2} + 12x\right) - \frac{3x^3}{4\times3}\right]_{-2}^{4}$

$= \left[\frac{3x^2}{4} + 6x - \frac{x^3}{4}\right]_{-2}^{4} = \frac{3\times4^2}{4} + 6\times4 - \frac{4^3}{4} - \left\{\frac{3\times4}{4} - 12 + \frac{8}{4}\right\}$

$= 12 + 24 - 16 - 3 + 12 - 2 = 27\ square\ units.$

or

Using integration find the area of triangular region whose sides have the equations :

$\boldsymbol{y = 2x + 1, y = 3x + 1\ and\ x = 4}$.

Solution : $y = 2x + 1$ ....(1)

$y = 3x + 1$ ....(2)

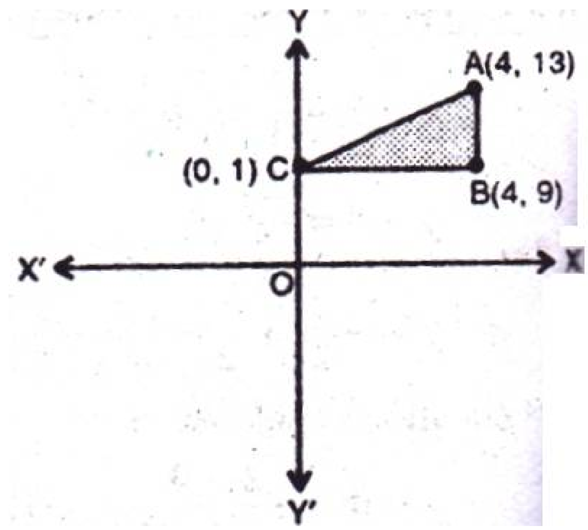
$x = 4$ ....(3)

The common pt. of lines (1) and (2) is

$$2x + 1 = 3x + 1 \Rightarrow x = 0$$

$\therefore$ *when* $x = 0,\ y = 1$

$\therefore$ (1) *and* (2) *has common pt.* $C(0,1)$.



Area of triangle ABC is given by

$= \int_0^4 (3x + 1)dx - \int_0^4 (2x + 1)dx$

$= \left[\frac{3x^2}{2} + x\right]_0^4 - \left[\frac{2x^2}{2} + x\right]_0^4$

$= (24 + 4q) - (16 + 4)$

$= 28 - 20$

$= 8\ sq.units.$

Q.30 **Find the shortest distance between the lines** **[5]**

$\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$ ***and*** $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$

Solution: $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$ and $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$

The equations in vector form are given as :

$\vec{r} = (-\hat{\imath} - \hat{\jmath} - \hat{k}) + \lambda(7\hat{\imath} - 6\hat{\jmath} + \hat{k})$

and $\vec{r} = (3\hat{\imath} + 5\hat{\jmath} + 7\hat{k}) + \mu(\hat{\imath} - 2\hat{\jmath} + \hat{k})$

On comparing with,

$\vec{r} = \overrightarrow{a_1} + \lambda\overrightarrow{b_1}$

and $\vec{r} = \overrightarrow{a_2} + \lambda\overrightarrow{b_2}$ we have

$\overrightarrow{a_1} = -\hat{\imath} - \hat{\jmath} - \hat{k},\ \overrightarrow{b_1} = 7\hat{\imath} - 6\hat{\jmath} + \hat{k}$

$\overrightarrow{a_2} = 3\hat{\imath} + 5\hat{\jmath} + 7\hat{k},\ \overrightarrow{b_2} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$

$\overrightarrow{a_2} - \overrightarrow{b_1} = 4\hat{\imath} + 6\hat{\jmath} + 8\hat{k}$

$\overrightarrow{a_1} - \overrightarrow{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & -6 & 1 \\ 1 & -2 & 1 \end{vmatrix}$

$= -4\hat{i} - 6\hat{j} + (-8)\hat{k} = -4\hat{i} - 6\hat{j} - 8\hat{k}$

$(\overrightarrow{b_1} \times \overrightarrow{b_2}) \cdot (\overrightarrow{a_2} - \overrightarrow{a_1}) = -16 - 36 - 64 = -116$

$|\overrightarrow{b_1} \times \overrightarrow{b_2}| = \sqrt{16 + 36 + 64} = \sqrt{116}$

$\therefore S.D. = \left|\frac{(\overrightarrow{b_1} \times \overrightarrow{b_2}) \cdot (\overrightarrow{a_2} - \overrightarrow{a_1})}{|\overrightarrow{b_1} \times \overrightarrow{b_2}|}\right| = \left|\frac{-116}{\sqrt{116}}\right| = \frac{116}{\sqrt{116}}$

$= \sqrt{116} = \sqrt{4 \times 29} = 2\sqrt{29}$ Units.

or

Find the equation of the plane that passes through three points $(1, 1, -1), (6, 4, -5)$ and $(-4, -2, 3)$.

Solution : We know that, the equation of plane passing through three non-collinear points is

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

By using given three points the equation of plane is

$$\begin{vmatrix} x - 1 & y - 1 & z - (-1) \\ 6 - 1 & 4 - 1 & -5 - (-1) \\ -4 - 1 & -2 - 1 & 3 - (-1) \end{vmatrix} = 0$$

or $$\begin{vmatrix} x - 1 & y - 1 & z + 1 \\ 5 & 3 & -4 \\ -5 & -3 & 4 \end{vmatrix} = 0$$

On operating $R_3 \to R_3 + R_{2,} we\ have$

$$\begin{vmatrix} x - 1 & y - 1 & z + 1 \\ 5 & 3 & -4 \\ 0 & 0 & 0 \end{vmatrix} = 0$$

$\Rightarrow \quad 0 = 0.$

Thus the plane is not fixed by the given three points, becaue the given three points are collinear.

Q.31 Solve the following linear programming problem graphically. [5]

Minimize $\quad Z = x + 2y$

Subject to the following constraints :

$$\mathbf{2x + y \geq 3,}$$
$$\mathbf{x + 2y \geq 6,}$$
$$\mathbf{x \geq 0, y \geq 0}$$

Solution: Here, objective function is

$Z = x + 2y$

Given constrains are

$2x + y \geq 3$ ....(i)

$x + 2y \geq 6$ ....(ii)

$x, y \geq 0$ ....(iii)

Converting inequation (i) and (ii) into equation, we get

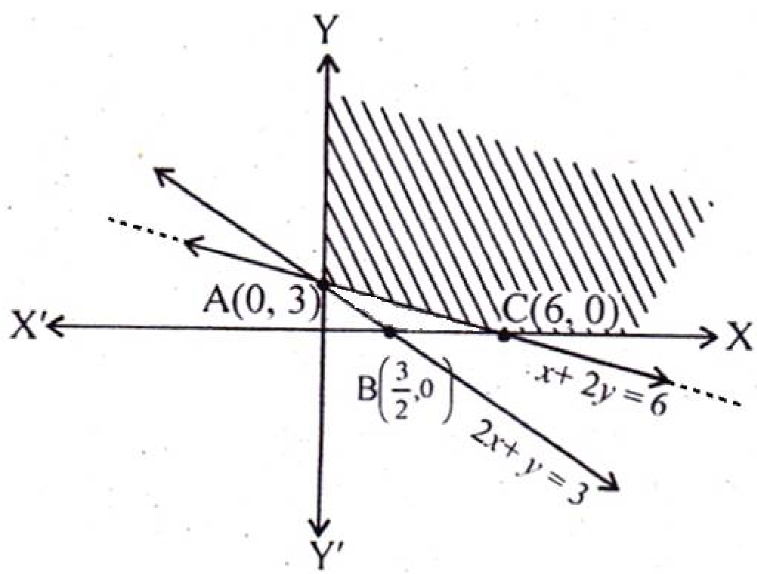
$2x + y = 3$ ....(iv)

and $x + 2y = 6$ ....(v)

Clearly equation (iv) represents al line through $A(0{,}3)$ and $B\left(\frac{3}{2}, 0\right)$

Also equation (v) represents a line through $A(0{,}3)$ and $C(6{,}0)$

And, in equation (iii) represents all non negatice values of x and y.



Now, put $(0, 0)$ in (i) and (ii), we get

from (i) $0 + 0 \geq 3$

$\Rightarrow 0 \geq 3$, which is false, so region away from $(0, 0)$ is solution of (i).

from (ii) $0 + 0 \geq 6$

$\Rightarrow 0 \geq 6$, which is false, so region away from $(0, 0)$ is solution of (ii).

Clearly, from graph we find that feasible region is unbounded.

Now, we will find minimum value of Z at corner points $A(0,3)$ and (6,0)

Corner point	$Z = x + 2y$
(0, 3)	6
(6,0)	6

Since, we obtain the sence value 6 at corner point A and C, hance we will obtain sance value 6 at the line joining $A(0, 3)$ and $C(6,0)$

Now, let $Z < 6 \Rightarrow x + 2y < 6$.

Now, we will draw the graph of $x + 2y < 6$.

Clearly $x + 2y < 6$ represents a dotted line through $A(0,3)$ and $C(6,0)$. Clearly feasible region and graph of $x + 2y < 6$ has no point common. Hence, 6 is minimum value of Z which occurs at all the point of line joining $A(0,3)$ and $C(6, 0)$.