Q.1 The principal value of $tan^{-1}(-\sqrt{3})$ is : [1]

(a) $\frac{\pi}{2}$ (b) $-\frac{\pi}{3}$

(c) $\frac{\pi}{3}$ (d) $-\frac{\pi}{2}$

Solution: (b)

Q.2 Let A be a nonsingular square matrix of order 3× 3. [1]

Then $|adj\ A|\ is$:

(a) $|A|$ (B) $|A|^3$

(C) $|A|^2$ (D) $|3A|$

Solution: (c)

Q.3 The derivative of $5^x is$: [1]

(a) 5^x (b) $\frac{5^x}{log5}$

(c) $5^x\ log\ 5$ (d) None of these

Solution: (c)

Q.4 On which of the following intervals is the function f given by $f(x) = x^{100} + sinx - 1$

Strictly decreasing? [1]

(a) (0, 1) (b) $\left(\frac{\pi}{2}, \pi\right)$

(C) $\left(0, \frac{\pi}{2}\right)$ (d) None of these

Solution : (d)

Q.5 $fe^x(f(x) + f'(x))dx\ is\ equal\ to$: [1]

(a) $e^x\ f'(x) + c$ (b) $e^x f(x) + c$

(c) $-e^x\ f'(x) + c$ (d) $-e^x\ f(x) + c$

Solution : (b)

Q.6 The degree of differential equation [1]

$\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$ is :

(a) 4 (b) 1

(c) **2** **(d)** **Not defined**

Solution : (b)

Q.7 **The vectors $\vec{a}$ and $\vec{b}$ are perpendicular if :** [1]

(a) $\vec{a}.\vec{b} = 0$ **(b)** $\vec{a}.\vec{b} \neq 0$

(c) $\vec{a} \times \vec{b} = 0$ **(d)** $\vec{a} \times \vec{b} \neq 0$

Solution : (a)

Q.8 **Find** $|\vec{a} - \vec{b}|, if |\vec{a}| = 2, |\vec{b}| = 3$ and $\vec{a}.\vec{b} = 4$ [1]

(a) $\sqrt{3}$ **(b)** $\sqrt{2}$

(c) $\sqrt{5}$ **(d)** $\sqrt{7}$

Solution : (c)

Q.9 **If a line makes angles $\frac{\pi}{2}, \frac{3\pi}{4}$ and $\frac{\pi}{4}$ with x, y, z – axis, respectively then direction consines of this line are :** [1]

(a) $\pm(1, 1, 1)$ **(b)** $\pm\left(0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

(c) $\pm\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$ **(d)** $\pm\left(0, \frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

Solution : (d)

Q.10 **If A and B are independent events, then :** [1]

(a) $P(A \cap B) = P(A).P(B)$

(b) $P(A \cup B) = P(A).P(B)$

(c) $P(A \cap B) = P(A) + P(B)$

(d) $P(A \cup B) = P(A) + P(B)$

Solution : (a)

Q.11 **Using elementary operations, find the inverse of matrix** $A = \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$ [2]

Solution : $A = \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$

We write A= IA

$\begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Operate $R_2 \to R_2 \to 2R_1$

$\begin{bmatrix}1 & 3\\0 & 1\end{bmatrix} = \begin{bmatrix}1 & 0\\-2 & 1\end{bmatrix} A$

Operate $R_1 \to R_1 \to 3R_2$

$\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} = \begin{bmatrix}7 & -3\\-2 & 1\end{bmatrix} A$

Thus $A^{-1} = \begin{bmatrix}7 & -3\\-2 & 1\end{bmatrix}$

Or

For matrix

$\boldsymbol{A} = \begin{bmatrix}\mathbf{1} & \mathbf{5}\\ \mathbf{6} & \mathbf{7}\end{bmatrix}$

Verify that $A - A'$ is a skew symmetric matrix.

Solution : We have, $A = \begin{bmatrix}1 & 5\\6 & 7\end{bmatrix} \Rightarrow A' = \begin{bmatrix}1 & 6\\5 & 7\end{bmatrix}$

Here, $A - A' = \begin{bmatrix}1 & 5\\6 & 7\end{bmatrix} - \begin{bmatrix}1 & 6\\5 & 7\end{bmatrix} = \begin{bmatrix}0 & -1\\1 & 0\end{bmatrix}$;

$(A - A')' = \begin{bmatrix}0 & 1\\-1 & 0\end{bmatrix} = -\begin{bmatrix}0 & -1\\1 & 0\end{bmatrix} = -(A - A')$

Which is a skew symmetric matrix.

Q.12 **Examine the function given by**

$\boldsymbol{f(x)} = \begin{cases}\frac{\boldsymbol{\sin x}}{\boldsymbol{x}}, \boldsymbol{x} < 0\\ \boldsymbol{x+1}, \boldsymbol{x \geq 0}\end{cases}$ $\boldsymbol{for\ continuity.}$ [2]

Solution : Given, $\boldsymbol{f(x)} = \begin{cases}\frac{\boldsymbol{\sin x}}{\boldsymbol{x}}, \boldsymbol{x} < 0\\ \boldsymbol{x+1}, \boldsymbol{x \geq 0}\end{cases}$

Here, L.H.L.= $\text{Lt}_{x\to 0^-} f(x) = \text{Lt}_{x\to 0^-} \frac{\sin x}{x} = \text{Lt}_{h\to 0} \frac{-\sin h}{-h} = 1$

Also, R.H.L. = $\text{Lt}_{x\to 0^+} f(x) = \text{Lt}_{x\to 0^+}(x+1) = \text{Lt}_{h\to 0}(0+h) + 1 = 1$

Clearly, L.H.L. = R.H.L.

Therefore, $\text{Lt}_{x\to 0} f(x)$ exists and is equal to 1.

Also, $f(0) = 1 = \text{Lt}_{x\to 0} f(x)$

Therefore, the function is continuous at $x = 0$.

Q.13 **A ballon which always remains spherical, has a variable diameter $\frac{3}{2}(2x+1)$. Find the rate of change of its volume with respect to x.** [2]

Solution: Let $'r'$ be radius and $'V'$be volume of balloon.

Diameter of the sphere = $\frac{3}{2}(2x+3)$

$\therefore$ Radius of the sphere (r) = $\frac{1}{2}\left[\frac{3}{2}(2x+3)\right] = \frac{3}{4}(2x+3)$

Volume of sphere (V) = $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\frac{27}{64}(2x+3)^3$

$$= \frac{9\pi}{16}(2x+3)^3$$

Rate of change of volume = $\frac{dV}{dx} = \frac{9\pi}{16}\cdot 3(2x+3)^2\cdot 2$

$$= \frac{27\pi}{8}(2x+3)^2$$

Q.14 **Form the differential equation of the family of hyperbolas having foci on x −axis and centre at origin.**

Solution: Equation of hyperbolia having focus on x −axis and centre at origin is **[2]**

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

Differentiating w.r.t., x we get

$$\frac{1}{a^2}\cdot 2x - \frac{1}{b^2}\cdot 2yy_1 = 0$$

$$\frac{yy_1}{b^2} = \frac{x}{a^2}$$

$$\frac{yy_1}{x^2} = \frac{b^2}{a^2}$$

Again differentiating w.r.t., x we get

$$\frac{x\frac{d}{dx}yy_1 - yy_1\frac{d}{dx}x}{x^2} = 0$$

$$x(yy_2 + y_1y_1) - yy_1 = 0$$

$$xyy_2 + x(y_1)^2 - yy_1 = 0$$

Q.15 **Find gof and fog, if $f(x) = 8x^3$ and $g(x) = x^{\frac{1}{3}}$** $\left[3\frac{1}{2}\right]$

Solution: Here, $f(x) = 8x^3$ and $g(x) = x^{\frac{1}{3}}$

Then $gof = g(f(x)) = g(8x^3) = (8x^3)^{\frac{1}{3}} = 2x$

and $fog = f(g(x)) = f\left(x^{\frac{1}{3}}\right) = 8(x^{\frac{1}{3}})^3 = 8x$.

Q.16 **Solve $2\tan^{-1}(\cos x) = \tan^{-1}(2\,cosec\,x)$** $\left[3\frac{1}{2}\right]$

Solution: We write, $2\tan^{-1}(\cos x) = \tan^{-1}(2\,cosec\,x)$

$\Rightarrow \tan^{-1}\left(\frac{2\cos x}{1-\cos^2 x}\right) = \tan^{-1}(2\,cosec\,x)$ $\left[using, 2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2}\right]$

$\Rightarrow \frac{2\cos x}{1-cos^2 x} = 2cosec\, x \Rightarrow \frac{2\cos x}{sin^2 x} = \frac{2}{\sin x} = 2\, cosec\, x$

$\Rightarrow 2\cos x = 2\sin x \quad \Rightarrow \quad cos^2 x = sin^2 x$

$\Rightarrow cos^2 x = 1 - cos^2 x \quad \Rightarrow \quad 2cos^2 x = 1$

$\Rightarrow \cos x = \frac{1}{\sqrt{2}} \Rightarrow x = \frac{\pi}{4}.$

Or

Express for following in the simplest form :

$$\boldsymbol{tan^1}\left(\frac{x}{\sqrt{a^2-x^2}}\right), |\boldsymbol{x}| < a$$

Solution: Let $x = a\sin\theta$, and $\frac{-\pi}{2} < \theta < \frac{\pi}{2}$

Now $\sqrt{a^2 - x^2} = \sqrt{a^2 - a^2\, sin^2\theta} = \sqrt{a^2 cos^2\theta}$

$= a\cos\theta$

$$\left[\because -\frac{\pi}{2} < \theta < \frac{\pi}{2} \Rightarrow \cos\theta > 0\right]$$

Hence $tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right) = tan^{-1}\left(\frac{a\sin\theta}{a\cos\theta}\right)$

$= tan^{-1}(\tan\theta)$

$= \theta = sin^{-1}\frac{x}{a}$

Q.17 **Prove that** $\begin{vmatrix} \boldsymbol{b+c} & \boldsymbol{a} & \boldsymbol{a} \\ \boldsymbol{b} & \boldsymbol{c+a} & \boldsymbol{b} \\ \boldsymbol{c} & \boldsymbol{c} & \boldsymbol{a+b} \end{vmatrix} = \boldsymbol{4\, abc}$ $\left[3\frac{1}{2}\right]$

Solution: Consider, $L.H.S. = \begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$

Operating $R_1 \to R_1 - R_2 - R_3$, we get

$$L.H.S. = \begin{vmatrix} 0 & -2c & -2b \\ b & c+a & b \\ c & c & a+b \end{vmatrix} = -2\begin{vmatrix} 0 & c & b \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

Again operating $R_2 \to R_2 - R_1,\ R_3 \to R_3 - R_1$, we get

$$L.H.S. = -2\begin{vmatrix} 0 & c & b \\ b & a & 0 \\ c & 0 & a \end{vmatrix} = -2[0 - c(ab - 0) + b(0 - ac)] = 4abc$$

Q.18 **Differentiate** $\sin(\{\boldsymbol{tan^{-1}(e^{-x})}\}$ **w.r.t. x** $\left[3\frac{1}{2}\right]$

Solution: Let $y = \sin(tan^{-1}e^{-x})$

Then $\frac{dy}{dx} = \frac{d}{dx}\ [\sin(tan^{-1}e^{-x})]$

$= \cos(tan^{-1}e^{-x}) \cdot \frac{d}{dx}\ (tan^{-1}e^{-x})$

$= \cos(tan^{-1}e^{-x}) \cdot \frac{1}{1+(e^{-x})^2} \cdot \frac{d}{dx}(e^{-x})$

$= \frac{\cos(tan^{-1}e^{-x})}{1-e^{-2x}} \cdot (-e^{-x}) = \frac{-e^{-x}\cdot\cos(tan^{-1}e^{-x})}{1+e^{-2x}}$.

Or

Find $\frac{dy}{dx}$ if $xy = e^{(x-y)}$

Solution: Given, that $xy = e^{(x-y)}$

Taking logarithm on both sides, we have

$\log x + \log y = \log e^{(x-y)} = x - y.$ (because, $\log e^{a} = a$)

Differentiating both sides w.r.t. x, we have

$\frac{1}{x} + \frac{1}{y}\frac{dy}{dx} = 1 - \frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx}\left(\frac{1}{y} + 1\right) = 1 - \frac{1}{x}$

$\Rightarrow \frac{dy}{dx} = \frac{1-\frac{1}{x}}{1+\frac{1}{y}} \quad or \quad \frac{dy}{dx} = \frac{y}{x}\left(\frac{x-1}{y-1}\right).$

Q.19 **Evaluate $\int \frac{5x}{(x+1)(x^2-4)}\ dx$** $\left[3\frac{1}{2}\right]$

Solution: Let $I = \int \frac{5x}{(x+1)(x^2-4)}\ dx = \int \frac{5x}{(x+1)(x-2)(x+2)}\ dx$

Since integrand is proper rational function, so we can decompose it into partial fraction.

That is, $I = \int \left(\frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x+2}\right) dx$ ....(i)

Where, $\frac{5x}{(x+1)(x-2)(x+2)} = \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x+2}$

$\Rightarrow\ 5x = A(x-2)(x+2) + B(x+1)(x+2) + C(x+1)(x-2)$ ....(ii)

Now, putting $x = -1$ in equation (ii), we get $A = \frac{5}{3}$

Putting, $x = 2$ in equation (ii), we get $B = \frac{5}{6}$

Putting, $x = -2$ in equation (ii), we get $C = \frac{-5}{2}$

Putting the values of A, B and C in equation (i), we get

$$I = \int \left(\frac{5/3}{x+1} + \frac{5/6}{x-2} - \frac{5/2}{x+2}\right) dx$$

$\Rightarrow \quad I = \frac{5}{3}\int \frac{dx}{x+1} + \frac{5}{6}\int \frac{dx}{x-2} - \frac{5}{2}\int \frac{dx}{x+2}$

$\Rightarrow \quad I = \frac{5}{3}\log|x+1| + \frac{5}{6}\log|x-2| - \frac{5}{2}\log|x+2| + c.$

Q.20 **Evaluate** $\int \frac{6x+7}{\sqrt{(x-5)(x-4)}}\, dx$ $\left[3\frac{1}{2}\right]$

Solution: Let $I = \int \frac{6x+7}{\sqrt{(x-5)(x-4)}}\, dx = \int \frac{6x+7}{\sqrt{x^2-9x+20}}\, dx$ ....(i)

Putting, $6x + 7 = A\frac{d}{dx}(x^2 - 9x + 20) + B$

$\Rightarrow 6x + 7 = A(2x - 9) + B$ ....(ii)

By equating the coefficients of x and constant terms, we get

$2A = 6$ and $B - 9A = 7 \quad \Rightarrow \quad A = 3$ and $B = 34$

Putting the values of A and B in equation (ii), we get

$6x + 7 = 3(2x - 9) + 34$

Now, putting the value of $6x + 7$ in equation (i), we get

$$I = \int \frac{3(2x-9)+34}{\sqrt{x^2-9x+20}}\, dx$$

$\Rightarrow \quad I = 3\int \frac{2x-9}{\sqrt{x^2-9x+20}}\, dx + 34\int \frac{dx}{\sqrt{x^2-9x+20}}$

$\Rightarrow \quad I = 3\, I_1 + 34 I_2$ ....(iii)

Consider, $I_1 = \int \frac{2x-9}{\sqrt{x^2-9x+20}}\, dx$

Putting, $x^2 - 9x + 20 = t \Rightarrow (2x - 9)dx = dt$

$\Rightarrow I_1 = \int \frac{dt}{\sqrt{t}} = \int t^{-1/2} + c_1 = 2t^{1/2} + c_1 \Rightarrow I_1 = 2\sqrt{x^2 - 9x + 20} + c_1$ ....(iv)

Now, consider $I_2 = \int \frac{dx}{\sqrt{x^2-9x+20}} \Rightarrow I_2 = \int \frac{dx}{\sqrt{\left(x-\frac{9}{2}\right)^2 - \left(\frac{1}{2}\right)^2}}$

$\Rightarrow \quad I_2 = \log\left|\left(x - \frac{9}{2}\right) + \sqrt{\left(x - \frac{9}{2}\right)^2 - \left(\frac{1}{2}\right)^2}\right| + c_2$

$\Rightarrow \quad I_2 = \log\left|x - \frac{9}{2} + \sqrt{x^2 - 9x + 20}\right| + c_2$ ....(v)

Substituting the values of I_1 and I_2 from equations (iv) and (v) in equation (iii), we get

$I = 6\sqrt{x^2 - 9x + 20} + 34\log\left|x - \frac{9}{2} + \sqrt{x^2 - 9x + 20}\right| + c;$

Where, $= 3c_1 + 34\, c_2$.

Q.21 **Evaluate** $\int_0^{\pi} \frac{x \sin x}{1+cos^2 x}\, dx$ $\left[3\frac{1}{2}\right]$

Solution : $\int_0^{\pi} \frac{x \sin x}{1+cos^2 x}\, dx.$ Then by $P_{4,}$ we have

I=$\int_0^{\pi} \frac{(\pi-x)\sin(\pi-x)}{1+cos^2(\pi-x)} dx = \pi \int_0^{\pi} \frac{sinx\, dx}{1+cos^2 x}$ –I

Or $2\,I = \int_0^{\pi} \frac{\sin x\, dx}{1+cos^2 x}$

Or I = $\frac{\pi}{2}\int_0^{\pi} \frac{\sin x\, dx}{1+cos^2 x}$

$Put \cos x = t\ so\ that - \sin x\, dx = dt. As\ x = 0, t = 1\ and\ as\ x = \pi, t = -1$

Therefore, (by P_1)$we\ get$

I = $\frac{-\pi}{2}\int_1^{-1} \frac{dt}{I+t^2} = \frac{-\pi}{2}\int_1^{-1} \frac{dt}{I+t^2}$

$= \pi \int_0^{1} \frac{dt}{I+t^2} \left(since\ \frac{1}{I+t^2} is\ even\ fuction\right)$

$= \pi[tan^{-1}]_0^1 = \pi[tan^{-1} - tan^{-1}0] = \pi\left[\frac{\pi}{4} - 0\right] = \frac{\pi^2}{4}$

Q.22 **Solve the differential equation:** $\left[3\frac{1}{2}\right]$

$(x + y)\frac{dy}{dx} = 1$

Solution : Given differential equation is (x+y)$\frac{dy}{dx} = 1 \Rightarrow \frac{dy}{dx} = \frac{1}{x+y}$

Taking its reciprocal, we get

$\frac{dx}{dy} = x + y \Rightarrow \frac{dx}{dy} - x = y$ .....(i)

Clearly, equation(i) is of the form $\frac{dx}{dy} + Rx = S$.

Here, R = -1, S = y

Now, I.F. = $e^{\int R dy} = e^{\int -dy} = e^{-y}$

Therefore, solution of equation (i) is x(I.F.) = $\int (I\,.F.) = \int (I.F.)S dy$

$\Rightarrow \quad x \cdot e^{-y} = \int e^{-y} \cdot y dy$

$\Rightarrow \quad x \cdot e^{-y} = -ye^{-y} - e^{-y} + c = e^{-y}(-y - I) + c$

$\Rightarrow \quad x = -y - I + ce^y$

$\Rightarrow x + y + I = ce^y$; which is required general solution.

Or

Solve the differential equation:

$$(x-y)dy-(x+y)dx=0$$

Solution : Given differential equation is (x − y)dy − (x + y)dx = 0 ....(i)

The differential equation (i) can be rewritten as

$$\frac{dy}{dx}=\frac{x+y}{x-y}=\frac{1+\frac{y}{x}}{1-\frac{y}{x}} \qquad ......(ii)$$

Which is a homogeneous equation.

Putting, $y=vx \Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$ $in\ equation(ii), we\ get$

$$v+x\frac{dv}{dx}=\frac{1+v}{1-v} \qquad \Rightarrow \qquad x\frac{dv}{dx}=\frac{1+v}{1-v}-v$$

$$\Rightarrow \quad x\frac{dv}{dx}=\frac{1+v^2}{1-v} \qquad \Rightarrow \qquad \frac{1-v}{1+v^2}dv=\frac{dx}{x}$$

On integrating, we get

$$\int\frac{1-v}{1+v^2}dv=\int\frac{dx}{x}$$

$$\Rightarrow \quad \int\frac{1}{1+v^2}dv-\frac{1}{2}\int\frac{2v}{1+v^2}dv=log|x|+C$$

$$\Rightarrow \quad tan^{-1}v-\frac{1}{2}log|1+v^2|=log|x|+C$$

$$\Rightarrow \quad 2\,tan^{-1}v-log\{(1+v^{2)}x^2\}=2C$$

$$\Rightarrow \quad 2\,tan^{-1}\left(\frac{y}{x}\right)-log\left\{\left(1+\frac{y^2}{x^2}\right)x^2\right\}=C_1$$

$\Rightarrow \quad 2\tan^{-1}\left(\frac{y}{x}\right)-\log(x^2+y^2)=C$, where c is an arbitrary constants;

which is the required general solution of given differential equation.

Q.23 **Find x if the four points $A(3,2,1), B(4,x,5), C(4,2,-2)$ and $D(6,5,-1)$ are coplanar.** $\left[3\frac{1}{2}\right]$

Solution : Here $\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}=\hat{\imath}+(x-2)\hat{\jmath}+4\hat{k}$

$$\overrightarrow{AC}=\overrightarrow{OC}-\overrightarrow{OA}=\hat{\imath}+0\hat{\jmath}-3\hat{k}$$

$$\overrightarrow{AD}=\overrightarrow{OD}-\overrightarrow{OA}=3\hat{\imath}+3\hat{\jmath}-2\hat{k}$$

If four points are coplanar then, $[\overrightarrow{AB}\ \overrightarrow{AC}\ \overrightarrow{AD}]$=0

$$\Rightarrow [\overrightarrow{AB}\ \overrightarrow{AC}\ \overrightarrow{AD}]=\begin{vmatrix}1 & x-2 & 4\\ 1 & 0 & -3\\ 3 & 3 & -2\end{vmatrix}=0$$

$$\Rightarrow (9)-(x-2)(-2+9)+4(3)=0$$

$\Rightarrow \quad 9 - 7(x - 2) + 12 = 0$

$\Rightarrow \quad 9 - 7x + 14 + 12 = 0$

$\Rightarrow \quad -7x + 35 = 0$

$\Rightarrow \quad x = 5$

Q.24 **Find the vector and Cartesian equations of plane that passes through the point $(5, 2, -4)$ and perpendicular to the line with direction ratios 2,3,-1.** $\left[3\frac{1}{2}\right]$

Solution : We have the position vector of point $(5,2,-4) as\ \vec{a} = \hat{5} + 2\hat{j} - 4\hat{k}$ and the normal vector $\vec{N}$

Perpendicular to the plane as $\vec{N} = 2\hat{\imath} + 3\hat{j} - \hat{k}$

Therefore, the vector equation of the plane is given by $(\vec{r} - \vec{a}).\vec{N} = 0$

Or $\left[\vec{r} - (5\hat{\imath} + 2\hat{j} - 4\hat{k})\right] \cdot \left(2\hat{\imath} + 3\hat{j} - \hat{k}\right) = 0$ ....(i)

Transforming (1) into Cartesian form, we have

$$\left[(x - 5)\hat{\imath} + (y - 2)\hat{j} + (z + 4)\hat{k}\right] \cdot \left(2\hat{\imath} + 3\hat{j} - \hat{k}\right) = 0$$

Or $\quad 2(x - 5) + 3(y - 2) - 1(z + 4) = 0$

i.e. $\quad 2x + 3y - z = 20$

which is the cartesian equation of the plane.

Q.25 **A die marked 1, 2, 3 in red and 4, 5, 6 in green is tossed. Let A be the event, 'the number is even' and B be the event, 'the number is red'. Are A and B independent?** $\left[3\frac{1}{2}\right]$

Solution : Since die has six faces, therefore the sample space is

$S = \{1,2,3,4,5,6\}$

Also, A : 'the number is even'

B: 'the number is red'

That is, A =$\{2,4,6\}$; $B = \{1,2,3\} and\ A \cap B = \{2\}$

Now, P(A)=$\frac{3}{6} = \frac{1}{2}; P(B) = \frac{3}{6} = \frac{1}{2}$

and P(A$\cap B) = \frac{1}{6} \neq \frac{1}{2} \times \frac{1}{2}$

$\Rightarrow P(A \cap B) \neq P(A)P(B)$

$\Rightarrow \quad A\ and\ B\ are\ not\ independent.$

Q.26 **If pair of dice is thrown 4 times. If getting a doublet is considered a sucess, find the probability of two successes.** $\left[3\frac{1}{2}\right]$

Solution : Here success is getting a doublet' and n = 4 (in case of Bernoullian trails)

When a pair of dice thrown once, then

$p = P(a\ success) = P(getting\ a\ doublet) = \frac{6}{36} = \frac{1}{6} and$

$q = P(a\ failure) = 1 - p = 1 - \frac{1}{6} = \frac{5}{6}$

Therefore, P (two successes) = ${}^4C_2(p)^2(q)^2 = \frac{4\times3}{2\times1}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^2$

$= 6 \times \frac{1}{36} \times \frac{25}{36} = \frac{25}{216}$.

Or

Give two independent events A and B such that $P(A) = 0.3, P(B) = 0.6$, find

(i) P (A and B)

(ii) P(A and not B)

Solution : (i) $P(A + B) = P(A) \times P(B) = 0.3 \times 0.6 = 0.18$

(ii) $P(A\ and\ not\ B) = P(A) \times P(notB)$

$= 0.3 \times [1 - P(B)]$

$= 0.3 \times (1 - 0.6)$

$= 0.3 \times 0.4 = 0.12$

Q.27 **Solve the following equations by Matrix Method:** **[5]**

$$\begin{aligned} 2x + 3y + 3z &= 5 \\ x - 2y + z &= -4 \\ 3x - y - 2z &= 3 \end{aligned}$$

Solution : Given system of equation is

$2x + 3y + 3z = 5, x - 2y + z = -4\ and\ 3x - y - 2z = 3.$

It can be written as AX = B;

Where A $\begin{bmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{bmatrix}, B = \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}.$

Now $|A| = \begin{vmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ -3 & -1 & -2 \end{vmatrix} = 2(4 + 1) - 3(-2 - 3) + 3(-1 + 6)$

$\Rightarrow |A| = 10 + 15 + 15 = 40 \neq 0$

Since $|A| \neq 0, therefore\ A^{-1} exists.$

Now, adj a = $\begin{vmatrix} 5 & 5 & 5 \\ 3 & -13 & 11 \\ 9 & 1 & -7 \end{vmatrix} = \begin{vmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{vmatrix}$

$and\ A^{-} = \frac{adj\ A}{|A|} = \frac{1}{40}\begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}$

Now, the solution of the given system of equations is given by

$x = A^{-1}B = \frac{1}{40}\begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}\begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix} = \frac{1}{40}\begin{bmatrix} 25 - 12 + 27 \\ 25 + 52 + 3 \\ 25 - 44 - 21 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{40}\begin{bmatrix} 40 \\ 80 \\ -40 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$

Therefore, x = 1 , y = 2, z = -1.

Q.28 **Show that the right circular cone of least curved surface and given volume has an altitude equal to $\sqrt{2}$ time the radius of the base.** **[5]**

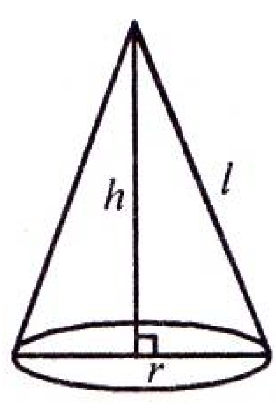
Solution : Let r be the radius, h be the straight height and l **be the slant height of the cone.**

Volume of cone (V) $=\frac{1}{3}\pi r^2 h$

$$h = \frac{3V}{\pi r^2}$$

Curved surface area of cone (S) $=\pi r l$

$$= \pi r\sqrt{r^2 + h^2} = \pi r\sqrt{r^2 + \frac{9V^2}{\pi^2 r^4}}$$



$S = \pi r\sqrt{\frac{\pi^2 r^6 + 9V^2}{\pi^2 r^4}}$

$S^2 = \pi^2 r^4 + 9V^2 r^{-2}$

Differentiate both side w.r.t.r.

$\frac{ds^2}{dr} = 4\pi^2r^3 - 18V^2r^{-3}$

$\frac{d^2s^2}{dr^2} = 12\pi^2r^2 + 54V^2r^{-4}$

Take $\frac{ds^2}{dr} = 0$

$4\pi^2r^3 - 18V^2r^{-3} = 0$

$4\pi^2r^3 = 18V^2r^{-3}$

$4\pi^2r^3 = \frac{18V^2}{r^3}$

$r^6 = \frac{9}{2\pi^2}.\frac{1}{9}\pi^2r^4h^2$ $\left[\because v = \frac{1}{3}\pi r^2h\right]$

$r^2 = \frac{h^2}{2}$

$r = \frac{h}{\sqrt{2}}$.

$\left.\frac{d^2s^2}{dr^2}\right] r = \frac{h}{\sqrt{2}} = 12\pi^2\frac{h^2}{2} + 54V^2\left(\frac{4}{h^4}\right) = 6\pi^2h^2 + 216\frac{V^2}{h^2} > 0$

∴ curved surface area is minimum when r = $\frac{h}{\sqrt{2}}$.

$\Rightarrow h = \sqrt{2}$ r

$\Rightarrow Altitude\ of\ cone\ is\ equal\ to\ \sqrt{2}\ times\ the\ radius\ of\ the\ base.$

Or

Find the point on the curve $y = x^3 - 11x + 5$ at which the tangent is : $y = x - 11$.

Solution : $y = x^3 - 11x + 5$

$\frac{dy}{dx} = 3x^2 - 11.$

Slope of tangent = $3x^2 - 11$.

Equation of tangent is $y = x - 11 \Rightarrow y - x + 11 = 0$

Slope= $-\frac{coeff.of\ x}{coeff.of\ y} = \frac{-1}{1} = 1.$

$\therefore\ 3x^2 - 11 = 1$

$\Rightarrow\ 3x^2 = 1 + 11$

$\Rightarrow\ 3x^2 = 12$

$\Rightarrow\ x^2 = 4$

$x = \pm 2.$

When $x = 2, y = (2)^3 - 11 \cdot 2 + 5 = 8 - 22 + 5$

$y = 13 - 22 = -9.$

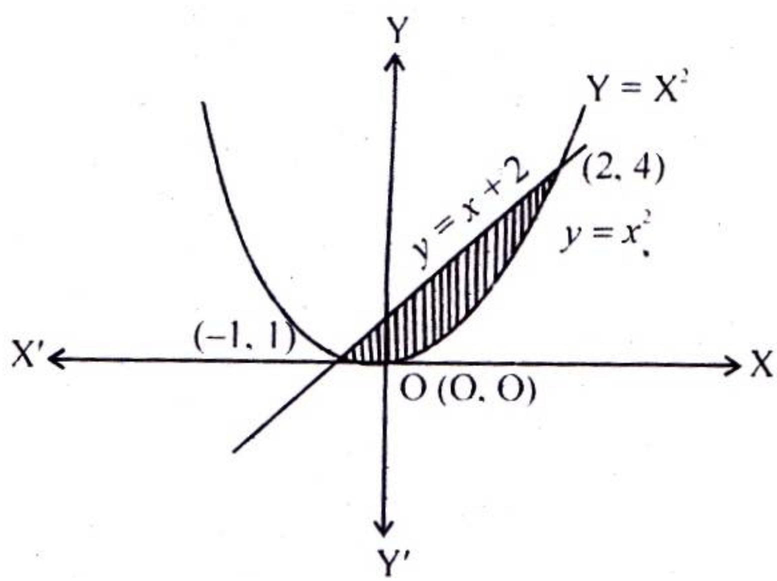
$\therefore Point\ (2, -9).$

Q.29 **Find the area of region bounded by the curves : $x^2 = y, the\ line\ y = x + 2$ and the x-axis.** **[5]**

Solution : Given parabola is y = x^2 ....(i)

and line is y = x + 2 ....(ii)

On solving (i) and (ii), the point of contact is (-1,1) and (2,-4)



$\therefore Required\ area\ is\ \int_{-1}^{2}(x + 2)dx - \int_{-1}^{2} x^2 dx$

$= \left[\frac{x^2}{2} + 2x\right]_{-1}^{2} - \left[\frac{x^3}{3}\right]_{-1}^{2} = 6 - \frac{1}{2} + 2 - 3 = \frac{9}{2}$ square units.

Or

Using integration find the area of region bounded by the triangle whose vertices are (-1,0), (1,3) And (3,2).

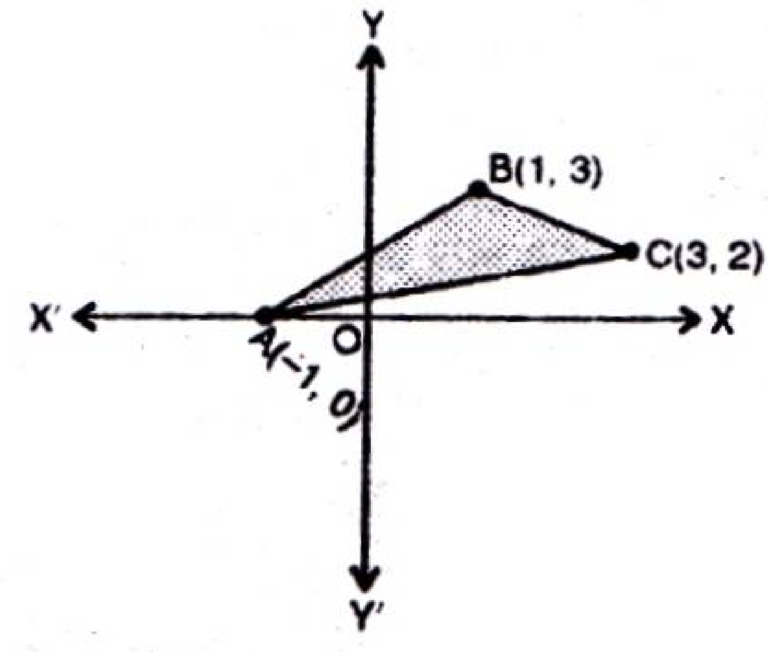
Solution : To find the area of triangle, we shall draw the rough sketch of the triangle.

Equation of lineAB is

$y - 0 = \frac{3-0}{1-(-1)}(x + 1)$ $\left[\because y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)\right]$

$y = \frac{3}{2}(x + 1).$

equation of side BC is



$y-3=\frac{2-3}{3-1}(x-1)$

$\Rightarrow y-3=\frac{-1}{2}(x-1)$

$\Rightarrow = -\frac{1}{2}x+\frac{1}{2}+3=-\frac{1}{2}x+\frac{7}{2}$

Equation of side AC is

$y-0=\frac{2-0}{3-(-1)}(x+1)$

$y=\frac{2}{4}(x+1)=\frac{1}{2}(x+1)$

Area of triangle

$=\int_{-1}^{1}\frac{3}{2}(x+1)dx+\int_{1}^{3}\left(\frac{-x+7}{2}\right)dx-\int_{-1}^{3}\frac{1}{2}(x+1)dx$

$=\frac{3}{2}\left(\frac{x^2}{2}+x\right)_{-1}^{1}+\left(\frac{\frac{-x^2}{2}+7x}{2}\right)_{1}^{3}-\frac{1}{2}\left(\frac{x^2}{2}+x\right)_{-1}^{3}$

$=\frac{3}{2}\left[\left(\frac{1}{2}+1\right)-\left(\frac{1}{2}-1\right)\right]+\frac{1}{2}\left[\left(\frac{-9}{2}+21\right)-\left(\frac{-1}{2}+7\right)\right]\frac{1}{2}\left[\left(\frac{3^2}{2}+3\right)-\left(\frac{(-1)^2}{2}+(-1)\right)\right]$

$=\frac{3}{2}\left[\frac{3}{2}+\frac{1}{2}\right]+\frac{1}{2}\left[\frac{33}{2}-\frac{13}{2}\right]-\frac{1}{2}\left[\frac{15}{2}+\frac{1}{2}\right]$

$=3+5-4=4\ sq.units.$

Q.30 **Find the shortest distance between the lines** **[5]**

$\vec{r} = (\hat{\imath} + 2\hat{\jmath} + \hat{k} + \lambda(\hat{\imath} - \hat{\jmath} + \hat{k})$ **and**

$\vec{r} = (2\hat{\imath} - \hat{\jmath} - \hat{k}) + \mu(2\hat{\imath} + \hat{\jmath} + 2\hat{k})$

Solution : Given equation of lines are,

$\vec{r} = (\hat{\imath} + 2\hat{\jmath} + \hat{k} + \lambda(\hat{\imath} - \hat{\jmath} + \hat{k})$ and ......(i)

and $\vec{r} = (2\hat{\imath} - \hat{\jmath} - \hat{k}) + \mu(2\hat{\imath} + \hat{\jmath} + 2\hat{k})$ ....(ii)

Comparing equation (i) and (i) with $\vec{r} = \vec{a}_2 + \lambda\vec{b}_2$ respectively , we get

$\vec{a}_1 = \hat{\imath} + \hat{2}\jmath + \hat{k}, \quad \overrightarrow{b_1} = \lambda(\hat{\imath} - \hat{\jmath} + \hat{k})$ and

$and \quad \overrightarrow{a_2} = 2\hat{\imath} - \hat{\jmath} - \hat{k}, \quad \overrightarrow{b_2} = 2\hat{\imath} + \hat{\jmath} + 2\hat{k}$

Now, $\overrightarrow{b_1} \times \overrightarrow{b_2} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix} = (-2-1)\hat{\imath} - (2-2)\hat{\jmath} + (1+2)\hat{k}$

$\Rightarrow \quad \overrightarrow{b_1} \times \overrightarrow{b_2} = -3\hat{\imath} + 3\hat{k}$

$\Rightarrow \quad |\overrightarrow{b_1} \times \overrightarrow{b_2}| = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}.$

Also, $\overrightarrow{a_2} - \overrightarrow{a_1} = (2\hat{\imath} - \hat{\jmath} - \hat{k}) - (\hat{\imath} + 2\hat{\jmath} + \hat{k}) = \hat{\imath} - 3\hat{\jmath} - 2\hat{k}$.

Therefore, shortest distance between given lines is

$$d = \left|\frac{(\overrightarrow{b_1} \times \overrightarrow{b_2}).(\overrightarrow{a_2} \times \overrightarrow{a_1})}{|\overrightarrow{b_1} \times \overrightarrow{b_2}|}\right| = \left|\frac{(-3\hat{\imath} + 3\hat{k}).(\hat{\imath} - 3\hat{\jmath} - 2\hat{k})}{3\sqrt{2}}\right|$$

$$\Rightarrow \quad d = \frac{|-3-6|}{3\sqrt{2}} = \frac{9}{3\sqrt{2}} = \frac{3}{\sqrt{2}}\ units.$$

Or

Find the equation of the plane that passes through three points (2,5,-3), (-2,-3,5) and (5,3,-3).

Solution : Let $\vec{a} = 2\hat{\imath} + 5\hat{\jmath} - 3\hat{k}, \vec{b} = -2\hat{\imath} - 3\hat{\jmath} + 5\hat{k}, \ \vec{c} = 5\hat{\imath} + 3\hat{\jmath} - 3\hat{k}$

Then the vector equation of the plane passing through $\vec{a}, \vec{b}$ and $\vec{c}$ and is given by

$$(\vec{r} - \vec{a}) \cdot (\overrightarrow{RS} \times \overrightarrow{RT}) = 0$$

Or $\quad (\vec{r} - \vec{a}) \cdot [(\vec{b} - \vec{a}) \times (\vec{c} - \vec{a})] = 0$

i.e. $\quad [\vec{r} - (2\hat{\imath} + 5\hat{\jmath} - 3\hat{k})] \cdot [(-4\hat{\imath} - 8\hat{\jmath} + 8\hat{k}) \times (3\hat{\imath} - 2\hat{\jmath})] = 0$

Q.31 **Solve the following linear programming problem graphically.** **[5]**

Minimize $Z = -3x + 4y$

Subject to the following constraints :

$x + 2y \leq 8$,

$3x + 2y \leq 12$,

$x \geq 0, y \geq 0$

Solution : Objective function $z = -3x + 4y$

Constraints are

$x + 2y \leq 8,\ 3x + 2y \leq 12,\ x \geq 0,\ y \geq 0$

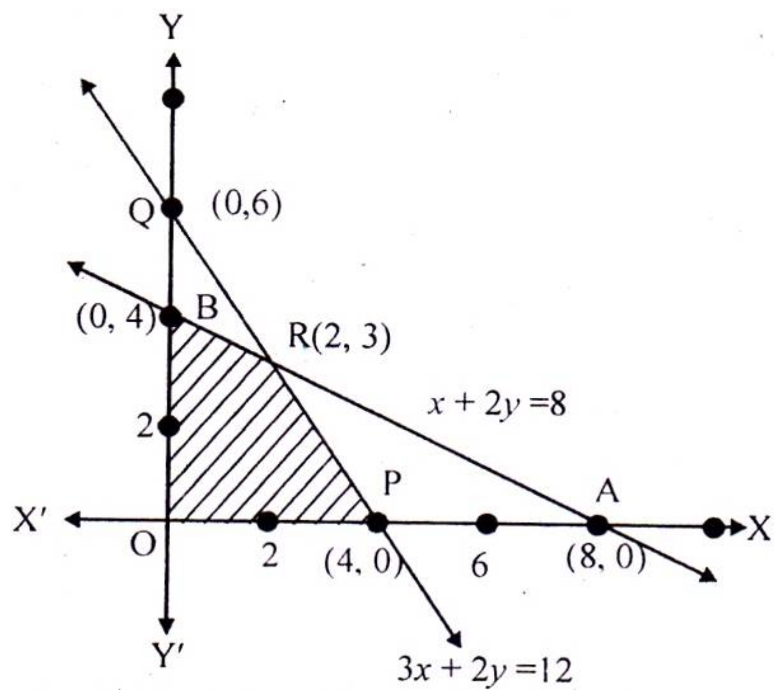
Consider the line $x + 2y = 8$

It pass through $A(8,0)$ *and* $B(0,4)$

Putting $x = 0,\ y = 0$ *in*

$x + 2y \leq 8, 0 \leq 8$ *which the true*

$\Rightarrow$ *region* $x + 2y \leq 8$ *lies on and below AB.*



Again the line $3x + 2y = 12$ passes through P(4,0), Q(0,6). Putting $x = 0, y = 0$ *in* $3x + 2y \leq 12$
$\Rightarrow$ $0 \leq 12$, *which is true*

$\therefore$ *Region* $3x + 2y \leq 12$ *lies on and belowPQ*

Here $x \geq 0$, $the\ region\ lies\ on\ and\ to\ the\ right\ of\ y - axis$

$and\ y \geq 0\ lies\ on\ and\ above\ x - axis$

On solving the equation $x + 2y = 8\ \ and\ 3x + 2y = 12$

We get $x = 2, y = 3 \Rightarrow R\ is\ (2,3) where\ AB\ snf\ PQ\ intersect\ the\ shaded\ region\ OPRB\ is\ the$

$feasible\ region.$

$At\ P(4,0)$ $\quad Z = -3x + 4y = -12 + 0 = -12$

$At\ R(2,3)$ $\quad Z = -6 + 12 = 6$

$At\ B(0,4)$ $\quad Z = 0 + 16 = 16$

$At\ Q(0,0)$ $\quad Z = 0$

Thus minimum value of Z is -12 at P(4,0) .